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## Review of 1-Ф AC Circuit Fundamentals



Series RLC circuit

$$
I \angle-\theta=\frac{V \angle 0}{R+j \omega L-\frac{j}{\omega C}} \quad \theta=\tan ^{-1}\left[\frac{\left(\omega L-\frac{1}{\omega C}\right)}{R}\right]
$$

## Review of 1-Ф AC Circuit Fundamentals(1)

$$
\mathrm{I} \angle-\theta=\mathrm{I} \operatorname{Cos} \theta-\mathrm{j} \mathrm{I} \operatorname{Sin} \theta
$$

$$
\text { Power factor }=\operatorname{Cos} \theta=\frac{\text { Re al Power }}{\text { Apparent Power }}
$$

Apparent Power =VI (multiply the rms value of input voltage and current (ignore phase angle))

Real Power $=I^{2}$ R (square of the rms current flowing through the reristor times the resistor (ignore phase angle))

Series Resonance occurs when $\omega \mathrm{L}-\frac{1}{\omega \mathrm{C}}=0$

$$
\begin{gathered}
\omega=\frac{1}{\sqrt{\mathrm{LC}}} \\
\mathrm{I}=\frac{\mathrm{V} \angle 0}{\mathrm{R}} \quad \text { is maximum in this case } \\
\text { Transformer }
\end{gathered}
$$

## Review of 1-Ф AC Circuit Fundamentals(3)



Parallel RLC circuit
Parallel Resonance occurs when $\omega \mathrm{L}-\frac{1}{\omega \mathrm{C}}=0$

$$
\begin{gathered}
\omega=\frac{1}{\sqrt{\mathrm{LC}}} \\
\mathrm{I}=\frac{\mathrm{V} \angle 0}{\mathrm{R}} \quad \text { is minimum in this case } \\
\text { Transformer }
\end{gathered}
$$

## The Transformer


(Primary has $\mathrm{N}_{1}$ turns)
(Secondary has $\mathrm{N}_{2}$ turns)

## The Transformer(2)

- The source side is called Primary
- The load side is called Secondary
- Ideally

1. The resistance of the coils are zero.
2. The relative permeability of the core in infinite.
3. Zero core or iron loss.
4. Zero leakage flux

## The Transformer(2)

i) Switch ' $S_{1}$ ' is closed and ' $S_{2}$ ' is open at $t=0$

The core does not have a flux at $\mathrm{t}=0$
We will now prove the following on the greenboard:
The voltage induced across each coil is proportional to its number of turns.

## The Transformer(3)

ii) Switch ' $\mathrm{S}_{2}$ ' is now closed

A current now starts to flow in resistance R . This current is $i_{2}(t)$ (flows out of the dotted terminal).

$$
\mathrm{i}_{2}(\mathrm{t})=\frac{\mathrm{e}_{2}(\mathrm{t})}{\mathrm{R}}=\frac{\mathrm{V} 2(\mathrm{t})}{\mathrm{R}}
$$

Thus a MMF $\mathrm{N}_{2} \mathrm{i}_{2}(\mathrm{t})$ is applied to the magnetic circuit. This will immediately make a current $\mathrm{i}_{1}(\mathrm{t})$ flow into the dot of the primary side, so that $\mathrm{N}_{1} \mathrm{i}_{1}(t)$ opposes $\mathrm{N}_{2} \mathrm{i}_{2}(\mathrm{t})$ and the original flux in the core remains constant. Otherwise, $\mathrm{N}_{2} \mathrm{i}_{2}(\mathrm{t})$ would make the core flux change drastically and the balance between $V_{1}$ and $e_{1}(t)$ will be disturbed.

## The Transformer(3)

We will now prove the following on the greenboard:

1) The current induced in each coil is inversely proportional to its number of turns.
2) Instantaneous input power to the transformer = Instantaneous output power from the transformer.

## The Transformer(3)

Observation: It was shown that the flux in the core is
$\Phi_{\mathrm{m}} \operatorname{Sin}(\omega \mathrm{t})$. Since the permeability of the core is infinite ideally zero current can produce this flux! In actuality, a current $I_{m}$, known as magnetizing current is required to setup the flux in the transformer. This current is within 5\% of the full load current in a well designed transformer.

$$
I_{m}=\frac{V 1 r m s}{\omega L_{1}} ; L_{1}=\frac{N_{1}^{2}}{\mathfrak{R}}
$$

$\mathrm{L}_{1}$ is the primary side self inductance.

## Transformer Example(1)


i) Find $\mathrm{I}_{1}, \mathrm{I}_{2}$ in the above transformer. Neglect magnetizing current.
ii) What is the reflected (referred) load impedance on the primary side
iii) If the resistance is replaced by a) 100 mH inductor b) $10 \mu \mathrm{~F}$ capacitance; what will be the reflected load impedance on the primary side?

## Transformer Example(1)

## Solution on greenboard

## Polarity (dot) convention

Terminals of different windings are of same polarity if currents entering (or leaving) them produce flux in the same direction in the core.


FIGURE 2.8 Polarity determination.

## How to check polarity?

1) Measure $e_{12}$ and $e_{34}$
2) Connect 2 and 4 and measure $e_{13}$
3) If $\mathrm{e}_{13}=\mathrm{e}_{12}+\mathrm{e}_{34}, 1$ and 4 have same polarity
4) If $\mathrm{e}_{13}=\mathrm{e}_{12}-\mathrm{e}_{34}, 1$ and 4 have different polarity

## Parallel operation of transformers


(a)

(b)

FIGURE 2.9 Parallel operation of single-phase transformers. (a) Correct connection. (b) Wrong connection.

Wrong connections give circulating between the windings that can destroy transformers.

## Transformer Equivalent circuit (1)



## Transformer Equivalent circuit (2)



## Transformer Equivalent circuit (3)



## Transformer Equivalent circuit (4)



## Open circuit Test

$\cdot$ It is used to determine $\mathrm{L}_{\mathrm{m} 1}\left(\mathrm{X}_{\mathrm{m} 1}\right)$ and $\mathrm{R}_{\mathrm{c} 1}$
-Usually performed on the low voltage side
-The test is performed at rated voltage and frequency under no load


FIGURE 2.12 No-load (or open-circuit) test. (a) Wiring diagram for open-circuit test (h) Eauivalent circuit under open circuit.

## Short circuit Test

-It is used to determine $\mathrm{Ll}_{\mathrm{p}}\left(\mathrm{X}_{\mathrm{eq}}\right)$ and $\mathrm{R}_{\mathrm{p}}\left(\mathrm{R}_{\mathrm{eq}}\right)$
-Usually performed on the high voltage side
-This test is performed at reduced voltage and rated frequency with the output of the low voltage winding short circuited such that rated current flows on the high voltage side.


FIGURE 2.13 Short-circuit test. (a) Wiring diagram for short-circuit test. (b) Equivalent circuit at short-circuit condition.

## Transformer Regulation

-Loading changes the output voltage of a transformer. Transformer regulation is the measure of such a deviation.

Definition of \% Regulation

$$
=\frac{\left|\mathrm{V}_{\text {no-load }}\right|-\left|\mathrm{V}_{\text {load }}\right|}{\left|\mathrm{V}_{\text {load }}\right|} * 100
$$

$\mathrm{V}_{\text {no-load }}=$ RMS voltage across the load terminals without load
$\mathrm{V}_{\text {load }}=\mathrm{RMS}$ voltage across the load terminals with a specified load

## Maximum Transformer Regulation



FIGURE 2.14 Voltage regulation.
$V_{1}=V_{2}{ }^{\prime} \angle 0^{0}+I_{2}{ }^{\prime} \angle \theta_{2}{ }^{0} \cdot Z_{\text {eq1 }} \angle \theta_{e q 1}{ }^{0}$
Clearly $V_{1}$ is max imum when

$$
\theta_{2}+\theta_{e q 1}=0 ; \text { or } \theta_{2}=-\theta_{e q 1}
$$

## Transformer Losses and Efficiency

-Transformer Losses

- Core/Iron Loss $=\mathrm{V}_{1}{ }^{2} / \mathrm{R}_{\mathrm{c} 1}$
- Copper Loss $=\mathrm{I}_{1}{ }^{2} \mathrm{R}_{1}+\mathrm{I}_{2}{ }^{2} \mathrm{R}_{2}$

Definition of \% efficiency

$$
\begin{gathered}
=\frac{V_{2} I_{2} \operatorname{Cos} \theta_{2}}{\operatorname{Losses}+V_{2} I_{2} \operatorname{Cos} \theta_{2}} * 100 \\
=\frac{V_{2} I_{2} \operatorname{Cos} \theta_{2}}{V_{1}^{2} / R_{c 1}+I_{1}^{2} R_{1}+I_{2}^{2} R_{2}+V_{2} I_{2} \operatorname{Cos} \theta_{2}} * 100 \\
=\frac{V_{2} I_{2} \operatorname{Cos} \theta_{2}}{V_{1}^{2} / R_{c 1}+I_{2}^{2} R_{e q 2}+V_{2} I_{2} \operatorname{Cos} \theta_{2}} * 100 \\
\operatorname{Cos} \theta_{2}=\text { load power factor }
\end{gathered}
$$

## Maximum Transformer Efficiency

The efficiency varies as with respect to 2 independent quantities namely, current and power factor
-Thus at any particular power factor, the efficiency is maximum if core loss = copper loss. This can be obtained by differentiating the expression of efficiency with respect to $I_{2}$ assuming power factor, and all the voltages constant.

- At any particular $\mathrm{I}_{2}$ maximum efficiency happens at unity power factor. This can be obtained by differentiating the expression of efficiency with respect to power factor, and assuming $I_{2}$ and all the voltages constant.
-Maximum efficiency happens when both these conditions are satisfied.



## Another Transformer Example

The following are the open circuit and short circuit test data of a single phase, $10 \mathrm{kVA}, 2200 / 220 \mathrm{~V}, 60 \mathrm{~Hz}$ transformer

|  | O/C Test (HV side <br> Open) | S/C Test (LV side <br> Shorted) |
| :---: | :---: | :---: |
| Voltmeter | 220 V | 150 V |
| Ammeter | 2.5 A | 4.55 A |
| Wattmeter | 100 W | 215 W |

i)Find the equivalent circuit with respect to HV and LV side
ii) Find the efficiency and regulation of the transformer when supplying rated load at 0.8 pf lag.
iii) Maximum efficiency and regulation.

## Transformer Example(2)

## Solution on greenboard

## Autotransformer


-Primary and secondary on the same winding. Therefore there is no galvanic isolation.

# Features of Autotransformer 

$\checkmark$ Lower leakage
$\checkmark$ Lower losses
$\checkmark$ Lower magnetizing current
$\checkmark$ Increase kVA rating
$\times$ No galvanic Isolation

## Autotransformer Theory and Example

Explained and worked out on Greenboard

## Review of balanced three phase circuits

- Two possible configurations: $\operatorname{Star}(\mathrm{Y})$ and delta ( $\Delta$ )
- Star has neutral, delta does not


## Star (Y) connection

-Line current is same as phase current
-Line-Line voltage is $\sqrt{ } 3$ phase-neutral voltage
-Power is given by $\sqrt{ } 3 \mathrm{~V}_{\mathrm{L}-\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos \theta$ or $3 \mathrm{~V}_{\mathrm{ph}} \mathrm{I}_{\mathrm{ph}} \cos \theta$

## Delta ( $\Delta$ ) connection

-Line-Line voltage is same as phase voltage
-Line current is $\sqrt{ } 3$ phase current
-Power is given by $\sqrt{3} \mathrm{~V}_{\mathrm{L}-\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos \theta$ or $3 \mathrm{~V}_{\mathrm{ph}} \mathrm{I}_{\mathrm{ph}} \cos \theta$

## Typical three phase transformer connections


(a) $Y$ - $\Delta$

(b) $Y-\Delta$

(e) $Y-Y$

FIGURE 2.17 Three-phase transformer connections.

## Other possible three phase transformer Connections

- Y- zigzag
$\bullet \Delta$ - zigzag
- Open Delta or V
-Scott or T


## How are three phase transformers made?

- Either by having three single phase transformers connected as three phase banks.
- Or by having coils mounted on a single core with multiple limbs
-The bank configuration is better from repair perspective, whereas the single three phase unit will cost less ,occupy less space, weighs less and is more efficient


## Phase-shift between line-line voltages in transformers


 phase transformer.

## Vector grouping of transformers

- Depending upon the phase shift of line-neutral voltages between primary and secondary; transformers are grouped. This is done for ease of paralleling. Usually transformers between two different groups should not be paralleled.
-Group 1 :zero phase displacement (Yy0, Dd0,Dz0)
- Group 2 : $180^{\circ}$ phase displacement (Yy6, Dd6,Dz6)
- Group $3: 30^{\circ}$ lag phase displacement (Dy1, Yd1,Yz1)
-Group $4: 30^{\circ}$ lead phase displacement (Dy11, Yd11,Yz11) ( $\mathrm{Y}=\mathrm{Y} ; \mathrm{D}=\Delta ; \mathrm{z}=\mathrm{zigzag}$ )


## Calculation involving 3-ph transformers


(a)

(b)

(c)

(d)

FIGURE 2.19 Three-phase transformer and equivalent circuit.

## An example involving 3-ph transformers


(a)

(b)


## Open - delta or V connection


(a)

(b)

FIGURE 2.20 V connection.

## Open - delta or V connection

Power from winding ' $a b$ '
is $\mathrm{P}_{\mathrm{ab}}=\mathrm{V}_{\mathrm{ab}} \mathrm{I}_{\mathrm{a}} \cos \left(30^{0}+\phi\right)$
Power from winding ' $b c$ '
is $\mathrm{P}_{\mathrm{cb}}=\mathrm{V}_{\mathrm{cb}} \mathrm{I}_{\mathrm{c}} \cos \left(30^{\circ}-\phi\right)$
Therefore total power is
$=2 \mathrm{~V}_{\mathrm{L}-\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos 30^{\circ} \cos \phi$ or $57.7 \%$ of total power from 3 phases

## Harmonics in 3- $\phi$ Transformer Banks

- In absence of neutral connection in a Y-Y transformers $3^{\text {rd }}$ harmonic current cannot flow
- This causes $3^{\text {rd }}$ harmonic distortion in the phase voltages (both primary and secondary) but not line-line voltages, as $3^{\text {rd }}$ harmonic voltages get cancelled out in line-line connections (see hw problem 2.22, where the voltage between the supply and primary neutrals is due to the third harmonic. This voltage can be modeled as a source in series with the fundamental voltage in the phase winding)
- Remedy is either of the following :
a) Neutral connections, b) Tertiary winding c) Use zigzag secondary d) Use star-delta or delta-delta type of transformers.
a) The phenomenon is explained using a star-delta transformer.


## Harmonics in 3-ф Transformer Banks(2)


(a)

(b)

FIGURE 2.23 Harmonic current in three-phase transformer connections. (a) Y- $\Delta$ connection. (b) Waveforms of exciting currents.

## Harmonics in 3-ф Transformer Banks(3)


(a)

(b)

(c)

FIGURE 2.24 Oscillograms of currents and voltages in a Y- $\Delta$-connected trans-
former.


FIGURE 2.25 Y-Y system with a tertiary ( $\Delta$ ) transformer.
Transformer

## Per-Unit (pu) System

$$
\text { -Quantity in pu }=\frac{\text { actual value of quantity }}{\text { base value of quantity }}
$$

- Values fall in a small zone and computational burden is less
-Easy to go from one side of a transformer to another without resorting to turns ratio multiplication and subsequent source of error
-Rated quantities ( voltage,current,power) are selected as base quantities.
-Losses, regulation etc. can also be defined in pu.


## Per-Unit (PU) System(2)

A single phase transformer is rated at $10 \mathrm{kVA}, 2200 / 220 \mathrm{~V}, 60 \mathrm{~Hz}$. Equivalent impedance referred to high voltage side is $10.4+\mathrm{j} 31.3 \Omega$. Find $\mathrm{I}_{\text {base }}, \mathrm{V}_{\text {base }}, \mathrm{P}_{\text {base }}, \mathrm{Z}_{\text {base }}$ on both sides. What is the pu equivalent impedance on both sides? If magnetizing current $\mathrm{I}_{\mathrm{m}}$ is 0.25 A on high voltage side what is it's value in pu?
-HV side;
$\mathrm{P}_{\text {base }}=10,000 \mathrm{VA}=1 \mathrm{pu}, \mathrm{V}_{\text {base }}=2200 \mathrm{~V}=1 \mathrm{pu}$
$\mathrm{I}_{\text {base }}=\mathrm{P}_{\text {base }} / \mathrm{V}_{\text {base }}=4.55 \mathrm{~A}=1 \mathrm{pu}$
$\mathrm{Z}_{\text {base }}=\mathrm{V}_{\text {base }} / \mathrm{I}_{\text {base }}=2200 / 4.55=483.52 \Omega=1 \mathrm{pu}$
$\mathrm{Z}_{\mathrm{eq}(\mathrm{pu})}=\mathrm{Z}_{\mathrm{eq}} / \mathrm{Z}_{\text {base }}=10.4+\mathrm{j} 31.3 / 483.52=0.0215+\mathrm{j} 0.0647 \mathrm{pu}$
$\mathrm{I}_{\mathrm{m}(\mathrm{pu})}=\mathrm{I}_{\mathrm{m}} / \mathrm{I}_{\text {base }}=0.25 / 4.55=0.055 \mathrm{pu}$

## Per-Unit (PU) System(3)

-LV side;
$\mathrm{P}_{\text {base }}=10,000 \mathrm{VA}=1 \mathrm{pu}, \mathrm{V}_{\text {base }}=220 \mathrm{~V}=1 \mathrm{pu}$
$\mathrm{I}_{\text {base }}=\mathrm{P}_{\text {base }} / \mathrm{V}_{\text {base }}=45.5 \mathrm{~A}=1 \mathrm{pu}$
$\mathrm{Z}_{\text {base }}=\mathrm{V}_{\text {base }} / \mathrm{I}_{\text {base }}=220 / 45.5=4.84 \Omega=1 \mathrm{pu}$
$\mathrm{Z}_{\mathrm{eq}(\mathrm{pu})=}=\mathrm{Z}_{\mathrm{eq}} / \mathrm{Z}_{\mathrm{base}}=0.104+\mathrm{j} 0.313 / 4.84=0.0215+\mathrm{j} 0.0647 \mathrm{pu}$
$\mathrm{I}_{\mathrm{m}(\mathrm{pu})}=\mathrm{I}_{\mathrm{m}} / \mathrm{I}_{\text {base }}=2.5 / 45.5=0.055 \mathrm{pu}$

## Transformer Construction



FIGURE 2.1 Transformer core construction. (a) Core-type, (b) Shell-type, (c) Lshaped lamination. (d) E-shaped lamination.

## Transformer Construction(2)



Figure 10.9a
Construction of a simple transformer.


Figure 10.9b
Stacking laminations inside a coil.

Left: Windings shown only on one leg Right: Note the thin laminations

## 3-ф Transformer Construction (3)


(a)

(b)

(c)


FIGURE 2.21 Development of a three-phase core-type transformer.

## 3-ф Transformer Construction(4)




Figure 10.19
Three-phase, type OA/FA/FOA transformer rated

Left: A $1300 \mathrm{MVA}, 24.5 / 345 \mathrm{kV}, 60 \mathrm{~Hz}$ transformer with forced oil and air (fan) cooling.
Right: A $60 \mathrm{MVA}, 225 / 26.4 \mathrm{kV}, 60 \mathrm{~Hz}$ showing the conservator.

