JYOTHISHMATHI INSTITUTE OF THECHNOLOGY AND SCIENCE
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FORMAL LANGUAGES AND AUTOMATA THEORY
TURING MACHINES

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Turing Machines are…

- Very powerful (abstract) machines that could simulate any modern day computer (although very, very slowly!)

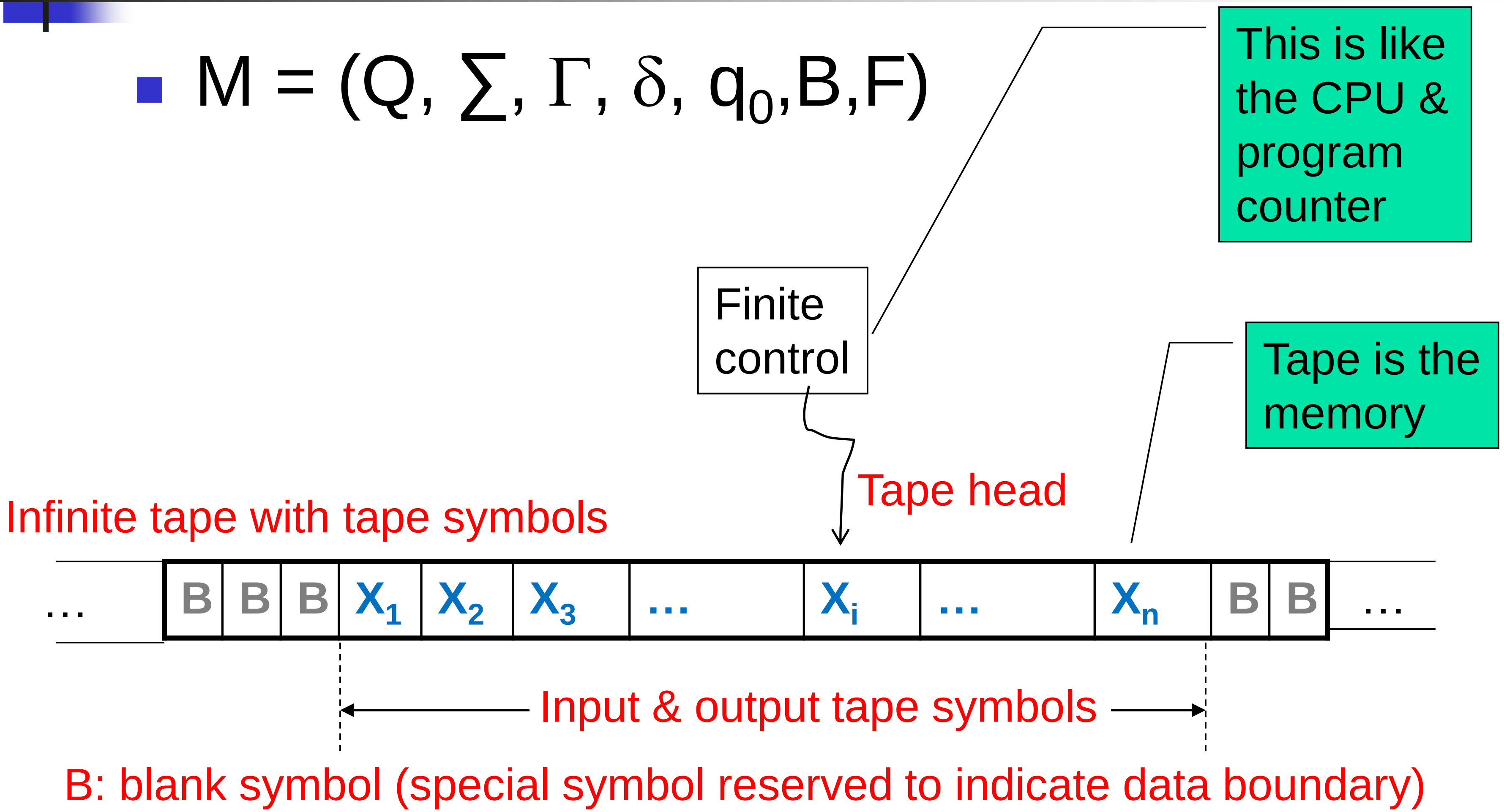
- Why design such a machine?
  - If a problem cannot be "solved" even using a TM, then it implies that the problem is ***undecidable***

For every input, answer YES or NO

- Computability vs. Decidability

A Turing Machine (TM)

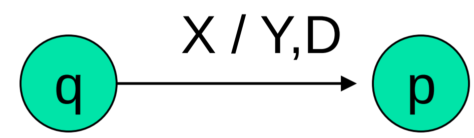
- $M = (Q, \sum, \Gamma, \delta, q_0, B, F)$



Transition function

You can also use:
➔ for R
🡐 for L

- One move (denoted by |---) in a TM does the following:
  - $\delta(q,X) = (p,Y,D)$
    - q is the current state
    - X is the current tape symbol pointed by tape head
    - State changes from q to p
    - After the move:
      - X is replaced with symbol Y
      - If D="L", the tape head moves "left" by one position.
        Alternatively, if D="R" the tape head moves "right" by one position.



ID of a TM

- Instantaneous Description or ID :
  - $X_1X_2\ldots X_{i-1}qX_iX_{i+1}\ldots X_n$
    means:
    - q is the current state
    - Tape head is pointing to X_i
    - $X_1X_2\ldots X_{i-1}X_iX_{i+1}\ldots X_n$ are the current tape symbols

- $\delta(q,X_i) = (p,Y,R)$ is same as:
  $X_1\ldots X_{i-1}qX_i\ldots X_n$ |---- $X_1\ldots X_{i-1}YpX_{i+1}\ldots X_n$
- $\delta(q,X_i) = (p,Y,L)$ is same as:
  $X_1\ldots X_{i-1}qX_i\ldots X_n$ |---- $X_1\ldots pX_{i-1}YX_{i+1}\ldots X_n$

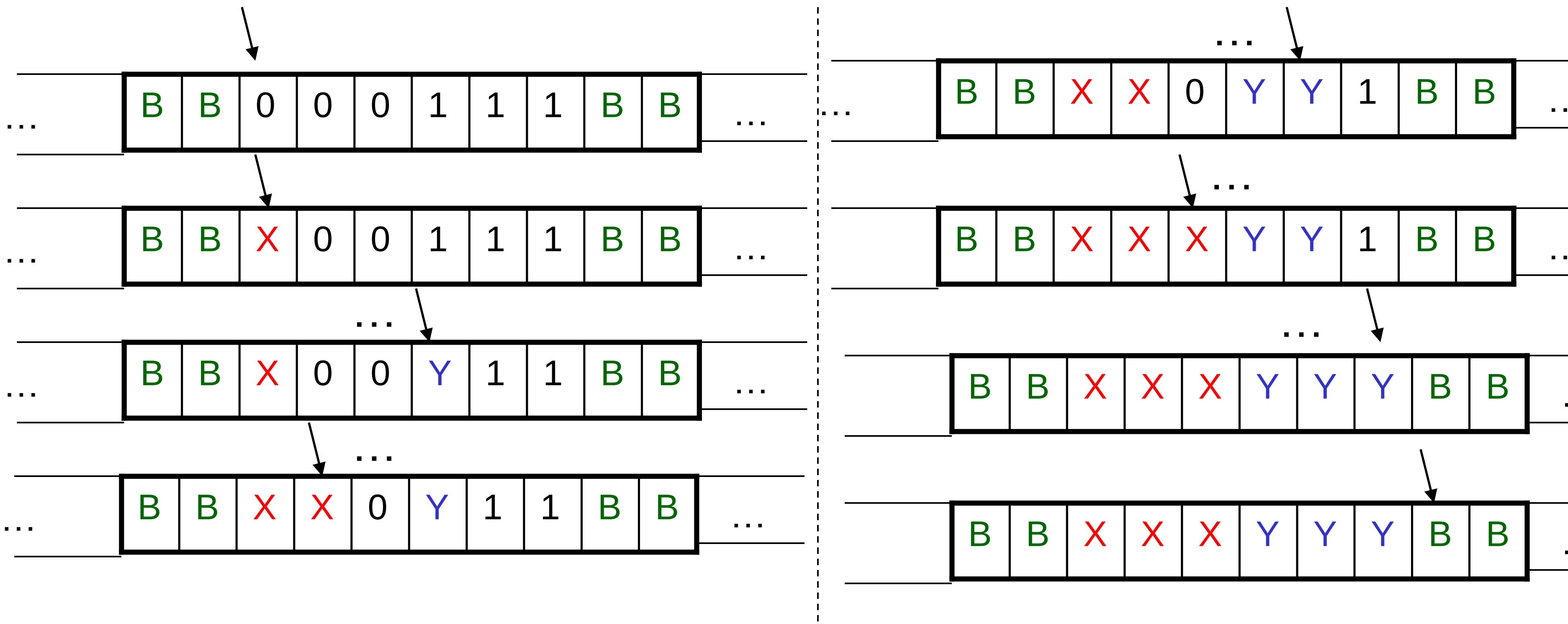
Way to check for Membership

- Is a string *w* accepted by a TM?

- <u>Initial condition:</u>
  - The (whole) input string *w* is present in TM, preceded and followed by infinite blank symbols
- <u>Final acceptance:</u>
  - Accept *w* if TM enters <u>final state</u> and halts
  - If TM halts and not final state, then reject

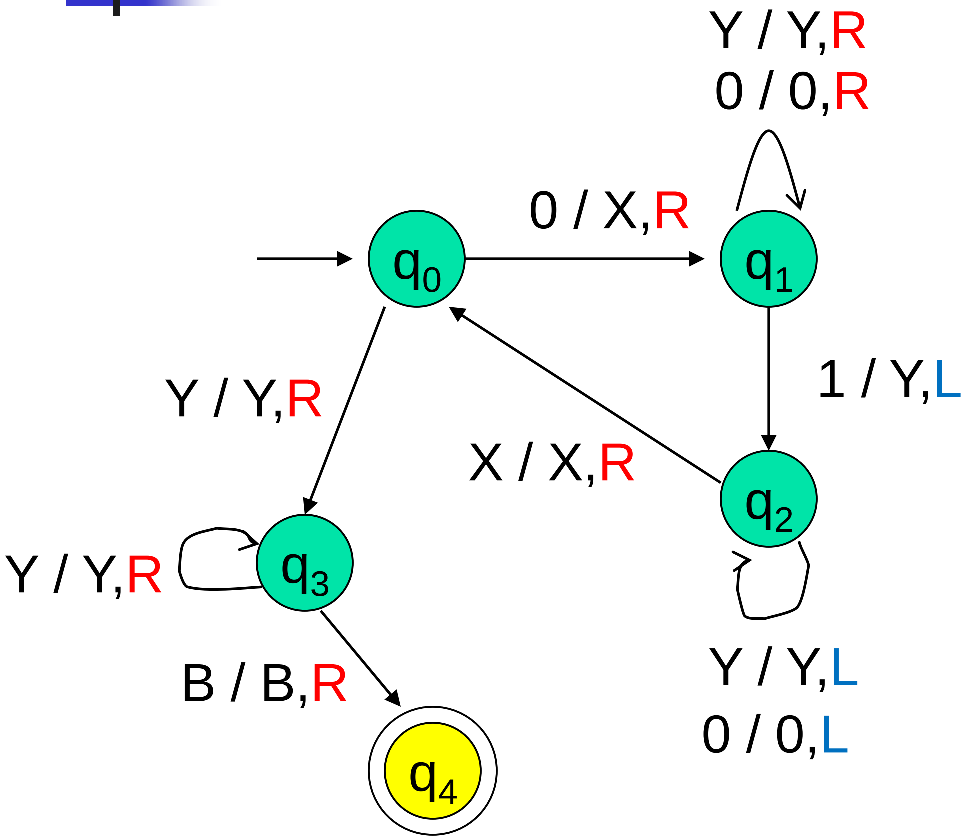
Example: $L = \{0^n1^n \mid n \geq 1\}$

- Strategy: w = 000111



Accept

TM for $\{0^n1^n \mid n \geq 1\}$



1. Mark next unread 0 with X and move right
2. Move to the right all the way to the first unread 1, and mark it with Y
3. Move back (to the left) all the way to the last marked X, and then move one position to the right
4. If the next position is 0, then goto step 1.
   Else move all the way to the right to ensure there are no excess 1s. If not move right to the next blank symbol and stop & accept.

*state diagram representation preferred

TM for $\{0^n1^n \mid n \geq 1\}$

	Next Tape Symbol				
Curr. State	0	1	X	Y	B
→ q_0	(q_1,X,R)	-	-	(q_3,Y,R)	-
q_1	$(q_1,0,R)$	(q_2,Y,L)	-	(q_1,Y,R)	-
q_2	$(q_2,0,L)$	-	(q_0,X,R)	(q_2,Y,L)	-
q_3	-	-	-	(q_3,Y,R)	(q_4,B,R)
$*q_4$	-	--	-	-	-

Table representation of the state diagram

TMs for calculations

- TMs can also be used for calculating values
  - Like arithmetic computations
  - Eg., addition, subtraction, multiplication, etc.