**JYOTHISHMATHI INSTITUTE OF TECHNOLOGY AND SCIENCE ,NUSTULAPUR,KARIMNAGAR**

# FLOW THROUGH PIPES

FLUID MECHANICS AND HYDRAULIC MACHINERY

S.CHANDRA SHEKHAR ASST.PROFF

II.B.TECH IISEM MECHANICAL

ACADEMIC YEAR 2018-2019

How big does the pipe have to be to carry a flow of x m3/s?

What will the pressure in the water distribution system be when a fire hydrant is open?

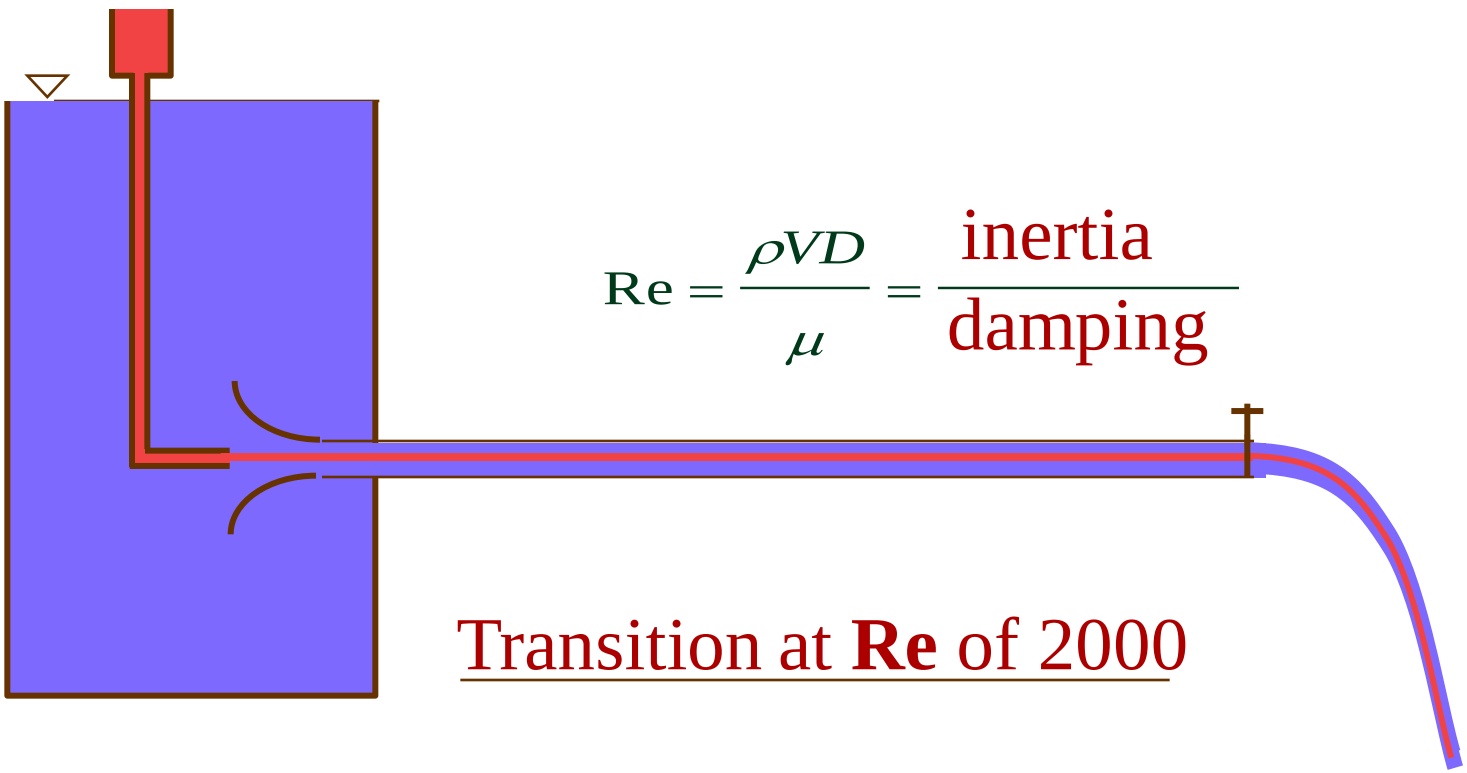
Can we increase the flow in this old pipe by adding a smooth liner?

# Viscous Flow in Pipes: Overview

- Boundary Layer Development
- Turbulence
- Velocity Distributions
- Energy Losses
  - Major
  - Minor
- Solution Techniques

# Laminar and Turbulent Flows

- Reynolds apparatus



$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{\text{inertia}}{\text{damping}}$$

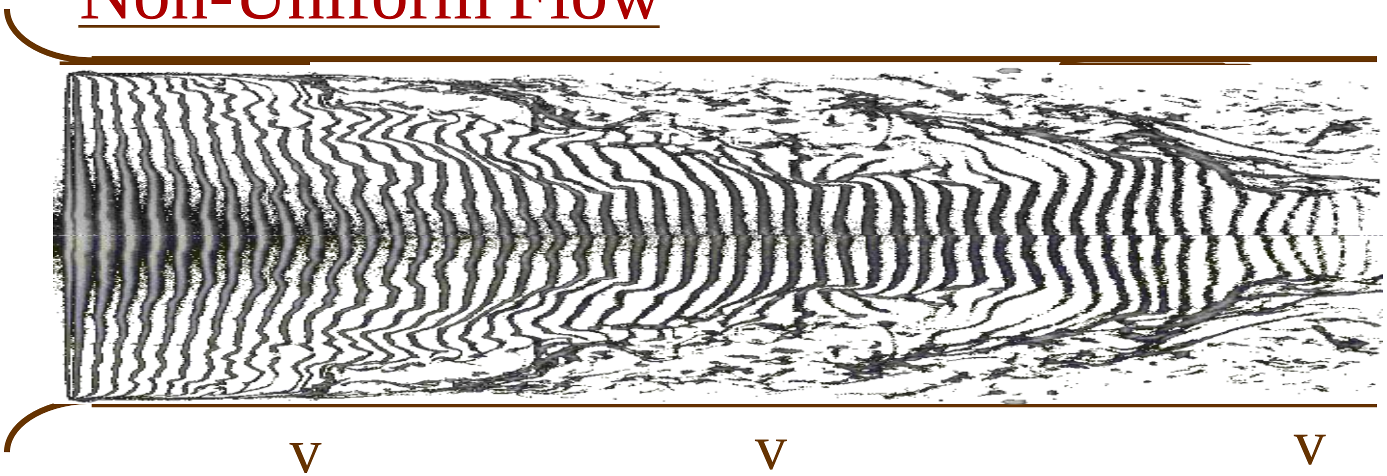
<u>Transition at **Re** of 2000</u>

# Boundary layer growth: Transition length

What does the water near the pipeline wall experience? Drag or shear

Why does the water in the center of the pipeline speed up? Conservation of mass

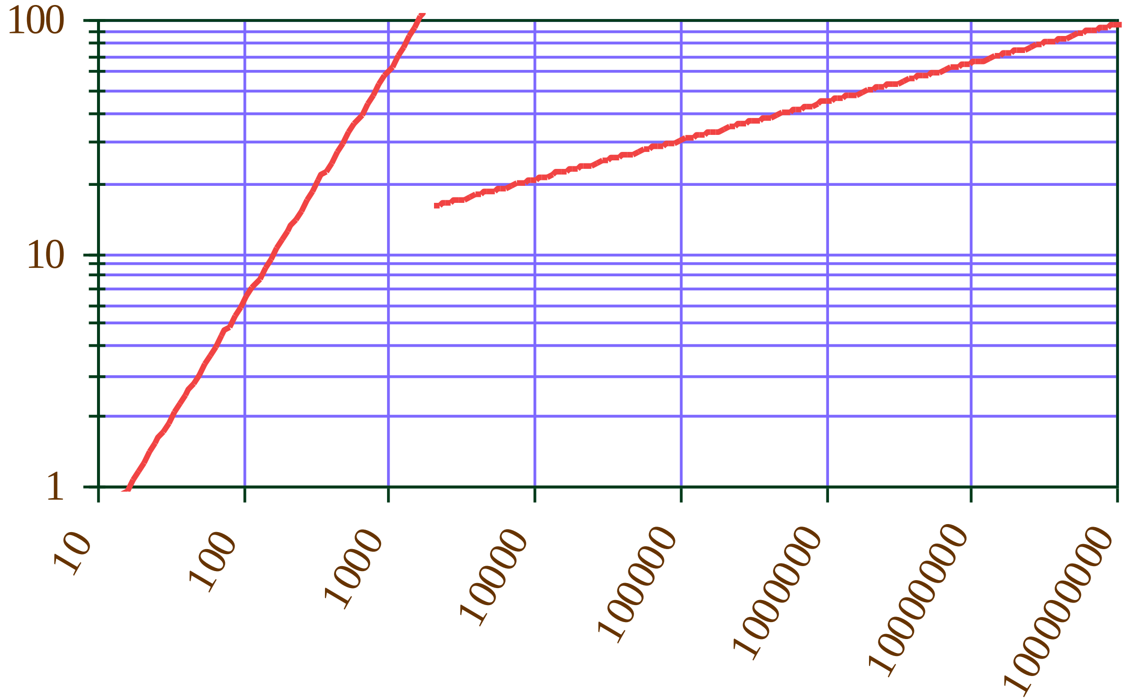
Non-Uniform Flow



# Entrance Region Length

$$\frac{l_e}{D} = f(\mathrm{Re}) \longrightarrow \frac{l_e}{D} = 0.06\,\mathrm{Re} \qquad \frac{l_e}{D} = 4.4(\mathrm{Re})^{1/6}$$

Distance for velocity profile to develop




Shear in the entrance region vs shear in long pipes?

# Velocity Distributions

- Turbulence causes transfer of momentum from center of pipe to fluid closer to the pipe wall.
- Mixing of fluid (transfer of momentum) causes the central region of the pipe to have relatively uniform velocity (compared to laminar flow)
- Close to the pipe wall, eddies are smaller (size proportional to distance to the boundary)

# Log Law for Turbulent, Established Flow, Velocity Profiles

$$\frac{u}{u_*} = 2.5 \ln \frac{y u_*}{\nu} + 5.0$$

Dimensional analysis and measurements

Valid for $\frac{y u_*}{\nu} > 20$

$$u_* = \sqrt{\frac{\tau_0}{\rho}}$$

Turbulence produced by shear!

Shear velocity

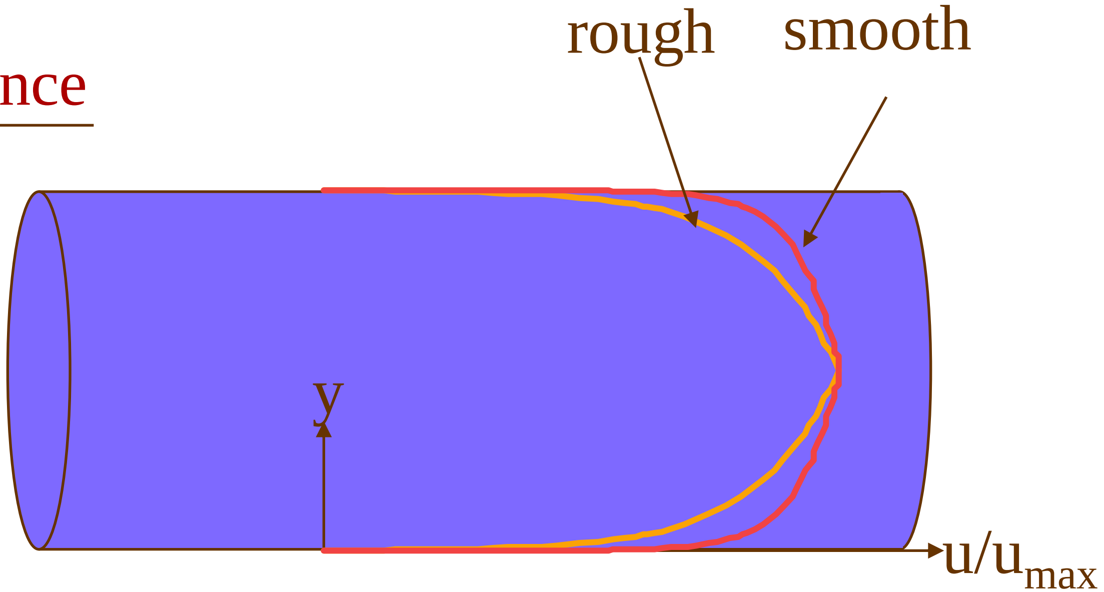
Velocity of large eddies

$$\tau_0 = \frac{\gamma h_f d}{4l}$$

Force balance

$$u_* = \sqrt{\frac{g h_f d}{4l}}$$

$$u_* = V\sqrt{\frac{f}{8}}$$



# Pipe Flow: The Problem

- We have the control volume energy equation for pipe flow
- We need to be able to predict the head loss term.
- We will use the results we obtained using dimensional analysis

# Viscous Flow: Dimensional Analysis

- Remember dimensional analysis?

$$C_p \frac{D}{l} = f\left(\frac{\varepsilon}{D}, \mathrm{Re}\right) \quad \text{Where} \quad \mathrm{Re} = \frac{\rho V D}{\mu} \quad \text{and} \quad C_p = \frac{-2\Delta p}{\rho V^2}$$

- Two important parameters!
  - Re - Laminar or Turbulent
  - ε/D - Rough or Smooth

- Flow geometry
  - internal in a bounded region (pipes, rivers): find $C_p$
  - external flow around an immersed object : find $C_d$

# Pipe Flow Energy Losses

$$\text{f} = \left( C_p \frac{D}{L} \right) = f\left( \frac{\varepsilon}{D}, \text{Re} \right)$$

Dimensional Analysis

$$\rho g h_l = -\Delta p - \rho g \Delta z$$

$$C_p = \frac{-2\Delta p}{\rho V^2} \qquad \rho g h_l = -\Delta p \qquad C_p = \frac{2 g h_l}{V^2}$$

More general

Assume horizontal flow

$$\text{f} = \frac{2 g h_\text{f}}{V^2} \frac{D}{L}$$

$$h_\text{f} = \text{f} \frac{L}{D} \frac{V^2}{2g}$$

Always true (laminar or turbulent)

Darcy-Weisbach equation

$$\text{f} = 8 \frac{u_*^2}{V^2} \qquad h_\text{f} = 8 \frac{L}{D} \frac{u_*^2}{2g}$$

# Friction Factor : Major losses

- Laminar flow
- Turbulent (Smooth, Transition, Rough)
- Colebrook Formula
- Moody diagram
- Swamee-Jain

# Laminar Flow Friction Factor

$$V = \frac{\rho g D^2}{32\mu}\frac{h_l}{L}$$

Hagen-Poiseuille

$$h_f = \frac{32\mu L V}{\rho g D^2}$$

$h_f \propto V$

$$Q = \frac{\pi D^4}{128\mu}\frac{\rho g h_l}{l}$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

Darcy-Weisbach

$$\frac{32\mu L V}{\rho g D^2} = f\frac{L}{D}\frac{V^2}{2g}$$

f independent of roughness!

$$f = \frac{64\mu}{\rho V D} = \frac{64}{Re}$$

Slope of -1 on log-log plot

# Turbulent Flow: Smooth, Rough, Transition

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

- Hydraulically smooth pipe law (von Karman, 1930)

$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{\mathrm{Re}\sqrt{f}}{2.51}\right)$$

- Rough pipe law (von Karman, 1930)

$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{3.7D}{\varepsilon}\right)$$

- Transition function for both smooth and rough pipe laws (Colebrook)

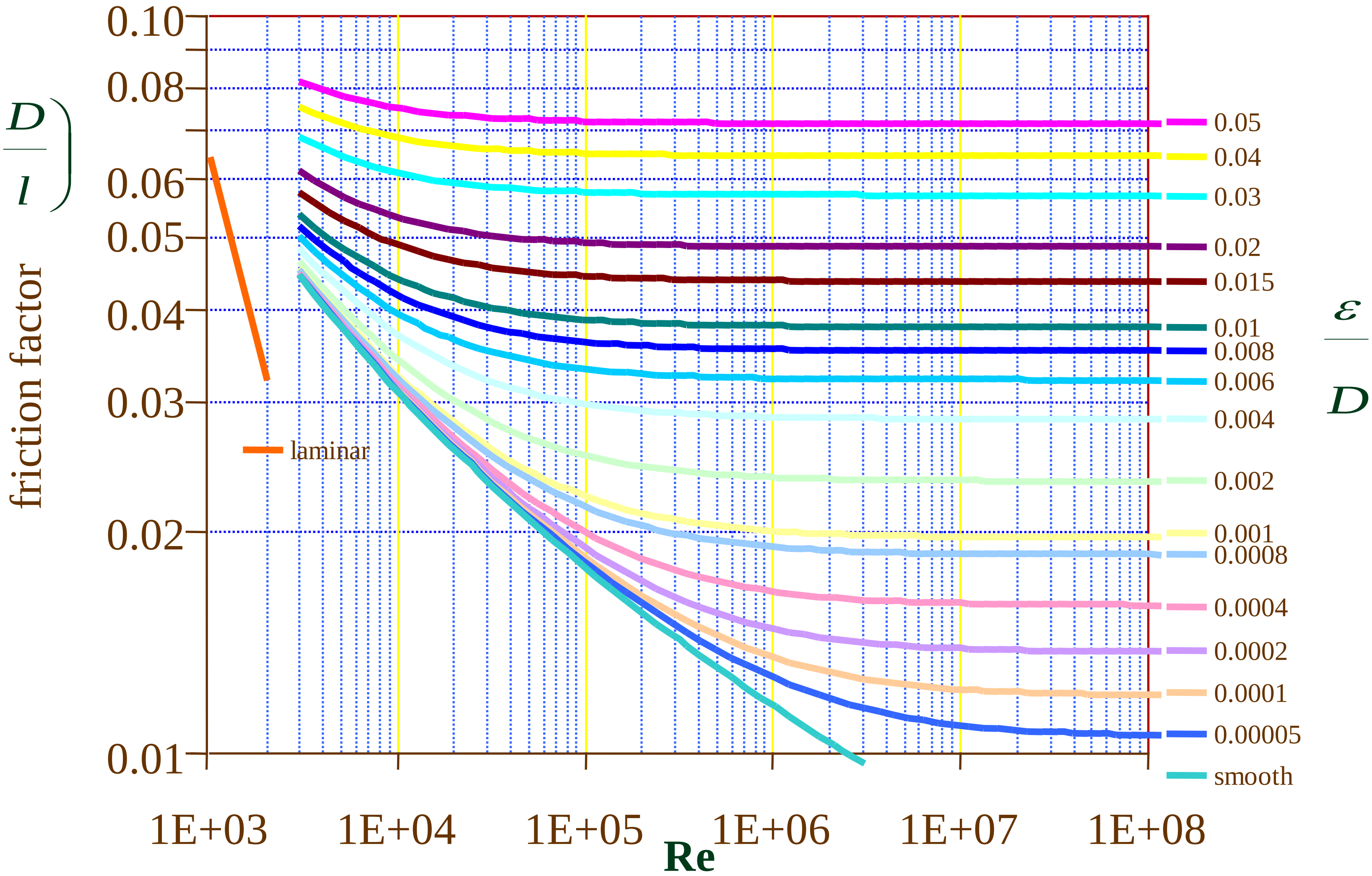
$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

$$u_* = V\sqrt{\frac{f}{8}}$$

(used to draw the Moody diagram)

# Moody Diagram

$$f = \left( C_p \frac{D}{l} \right)$$



# Swamee-Jain

- 1976
- limitations
  - ε/D < 2 x $10^{-2}$
  - Re >3 x $10^3$
  - less than 3% deviation from results obtained with Moody diagram
- easy to program for computer or calculator use

$$f = \frac{0.25}{\left[\log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2}$$

no f

$$Q = \frac{-\pi}{\sqrt{2}} D^{5/2} \sqrt{\frac{gh_f}{L}} \log\left(\frac{\varepsilon}{3.7D} + 2.51\nu\sqrt{\frac{L}{2gh_f D^3}}\right)$$

Colebrook

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{LQ^2}{gh_f}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{gh_f}\right)^{5.2}\right]^{0.04}$$

Each equation has two terms. Why?

# Pipe roughness

| pipe material | pipe roughness ε (mm) |
|---|---|
| glass, drawn brass, copper | 0.0015 |
| commercial steel or wrought iron | 0.045 |
| asphalted cast iron | 0.12 |
| galvanized iron | 0.15 |
| cast iron | 0.26 |
| concrete | 0.18-0.6 |
| rivet steel | 0.9-9.0 |
| corrugated metal | 45 |
| PVC | 0.12 |

$\frac{\varepsilon}{d}$ Must be dimensionless!

# Solution Techniques

- find head loss given (D, type of pipe, Q)

$$\mathrm{Re} = \frac{4Q}{\pi D \nu} \qquad \mathrm{f} = \frac{0.25}{\left[\log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \qquad h_{\mathrm{f}} = \mathrm{f}\frac{8}{\pi^2 g}\frac{LQ^2}{D^5}$$

- find flow rate given (head, D, L, type of pipe)

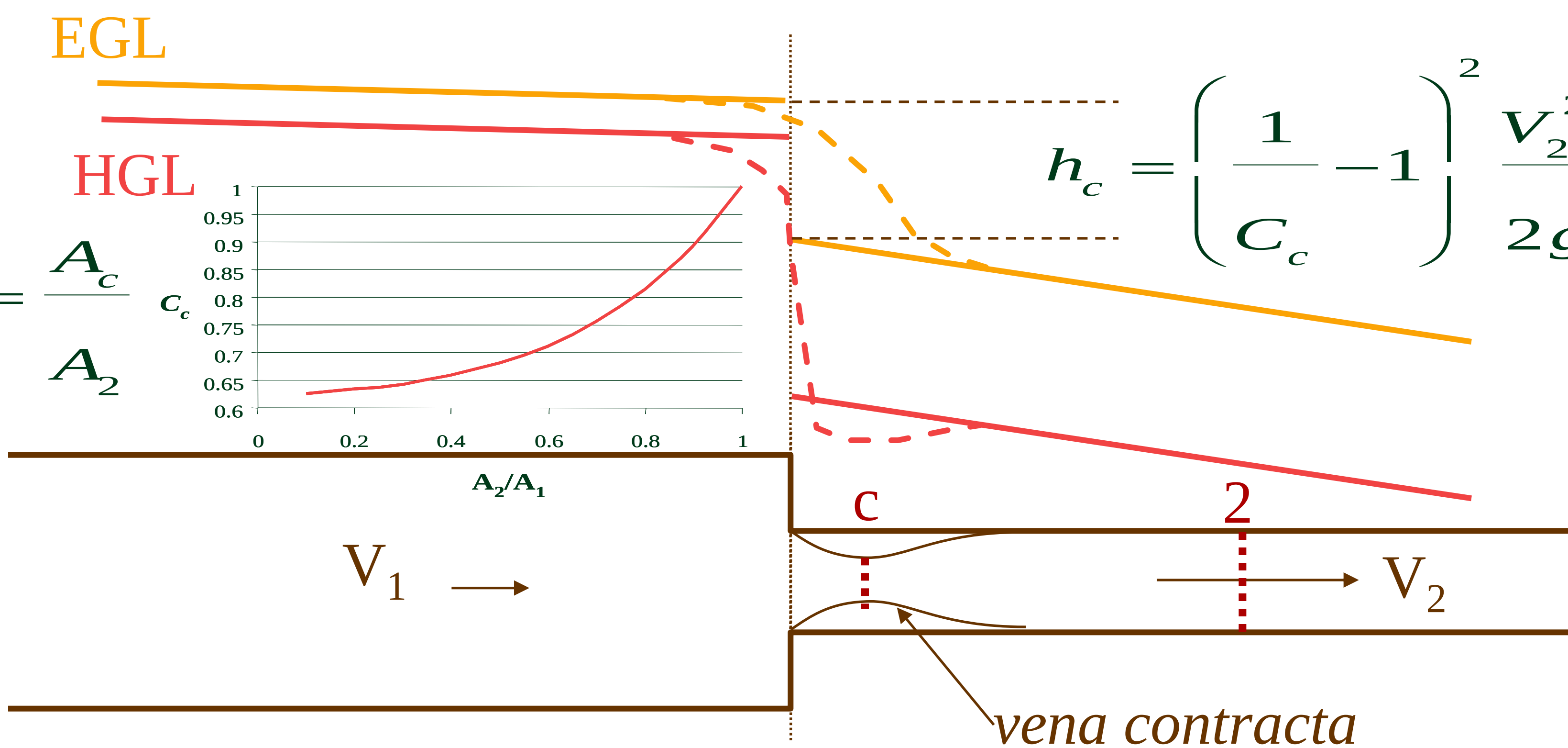
$$Q = \frac{-\pi}{\sqrt{2}} D^{5/2} \sqrt{\frac{g h_{\mathrm{f}}}{L}} \log\left(\frac{\varepsilon}{3.7D} + 2.51\nu\sqrt{\frac{L}{2 g h_{\mathrm{f}} D^3}}\right)$$

- find pipe size given (head, type of pipe,L, Q)

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{LQ^2}{g h_{\mathrm{f}}}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{g h_{\mathrm{f}}}\right)^{5.2}\right]^{0.04}$$

$$h_{ex} = \frac{V_{in}^2}{2g}\left(1 - \frac{A_{in}}{A_{out}}\right)^2$$

# Sudden Contraction




$$C_c = \frac{A_c}{A_2}$$

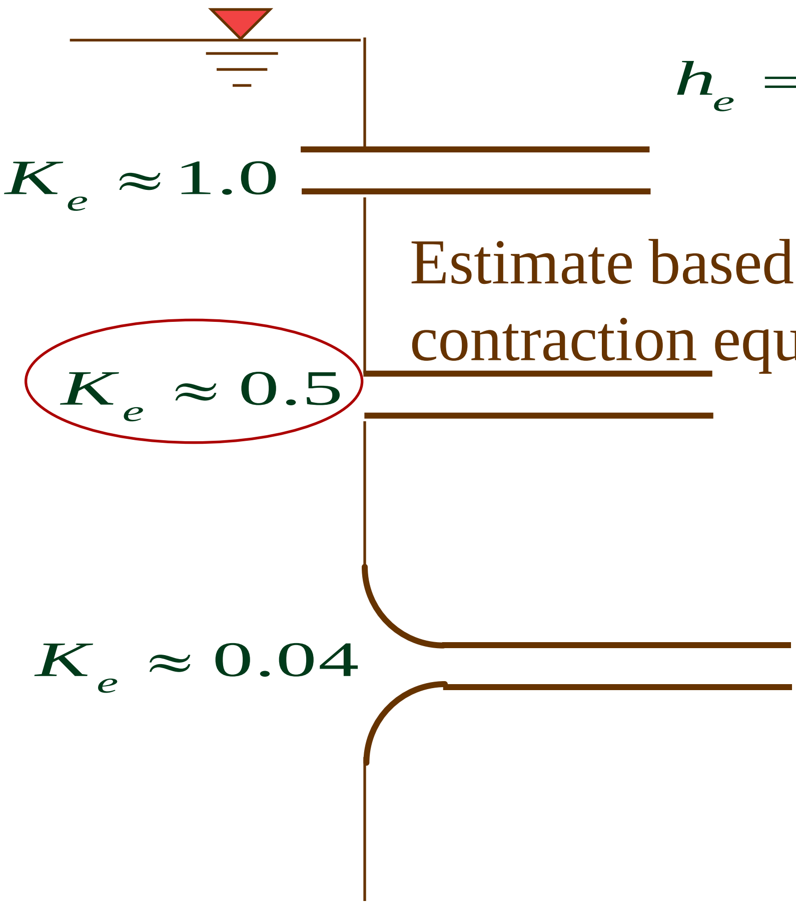
$$h_c = \left(\frac{1}{C_c} - 1\right)^2 \frac{V_2^2}{2g}$$

- Losses are reduced with a gradual contraction
- Equation has same form as expansion equation!

# Entrance Losses

$$h_e = K_e \frac{V^2}{2g}$$

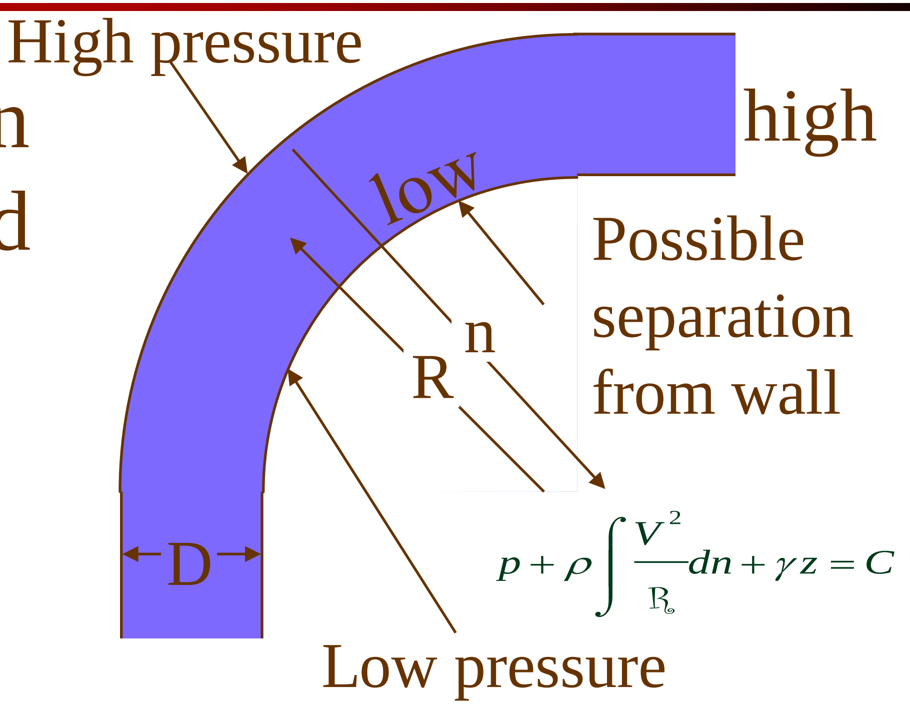
Losses can be reduced by accelerating the flow gradually and eliminating the vena contracta




Estimate based on contraction equations!

# Head Loss in Bends

- Head loss is a function of the ratio of the bend radius to the pipe diameter (R/D)
- Velocity distribution returns to normal several pipe diameters downstream




$$p+\rho\int \frac{V^2}{R}\,dn+\gamma z=C$$

$$h_b = K_b \frac{V^2}{2g}$$

<u>$K_b$ varies from 0.6 - 0.9</u>

# Head Loss in Valves

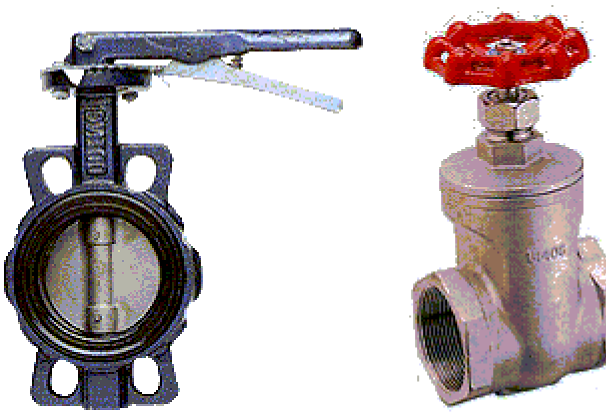
- Function of valve type and valve position
- The complex flow path through valves can result in high head loss (of course, one of the purposes of a valve is to create head loss when it is not fully open)

What is V?

$$h_v = K_v \frac{V^2}{2g}$$

$$h_v = K_v \frac{8Q^2}{g\pi^2 D^4}$$

Can $K_v$ be greater than 1? ___Yes!___



# Solution Techniques

- Neglect minor losses
- ~~Equivalent pipe lengths~~
- Iterative Techniques
  - Using Swamee-Jain equations for D and Q
  - Using Swamee-Jain equations for head loss
  - Assume a friction factor
- Pipe Network Software

# Solution Technique: Head Loss

- Can be solved explicitly

$$h_{minor} = \sum K \frac{V^2}{2g}$$

$$h_{minor} = \frac{8Q^2}{g\pi^2} \sum \frac{K}{D^4}$$

$$\mathrm{Re} = \frac{4Q}{\pi D \nu}$$

$$f = \frac{0.25}{\left[\log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2}$$

$$h_f = \mathrm{f}\,\frac{8}{g\pi^2}\frac{LQ^2}{D^5}$$

$$h_l = \sum h_f + \sum h_{minor}$$

# Find D or Q
# Solution Technique 1

- Assume all head loss is major head loss
- Calculate D or Q using Swamee-Jain equations
- Calculate minor losses $h_{ex} = K\frac{8Q^2}{g\pi^2 D^4}$
- Find new major losses by subtracting minor losses from total head loss $h_f = h_l - \sum h_{ex}$

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{LQ^2}{gh_f}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{gh_f}\right)^{5.2}\right]^{0.04}$$

$$Q = \frac{-\pi}{\sqrt{2}} D^{5/2}\sqrt{\frac{gh_f}{L}}\log\left(\frac{\varepsilon}{3.7D} + 2.51\nu\sqrt{\frac{L}{2gh_f D^3}}\right)$$

# Find D or Q
# Solution Technique 2: Solver

- Iterative technique
- Solve these equations

$$\mathrm{Re} = \frac{4Q}{\pi D \nu} \qquad \mathrm{f} = \frac{0.25}{\left[\log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \qquad h_\mathrm{f} = \mathrm{f}\,\frac{8}{g\pi^2}\frac{LQ^2}{D^5}$$

$$h_{minor} = K\frac{8Q^2}{g\pi^2 D^4}$$

$$h_l = \sum h_\mathrm{f} + \sum h_{minor}$$

Use goal seek or Solver to find discharge that makes the calculated head loss equal the given head loss.

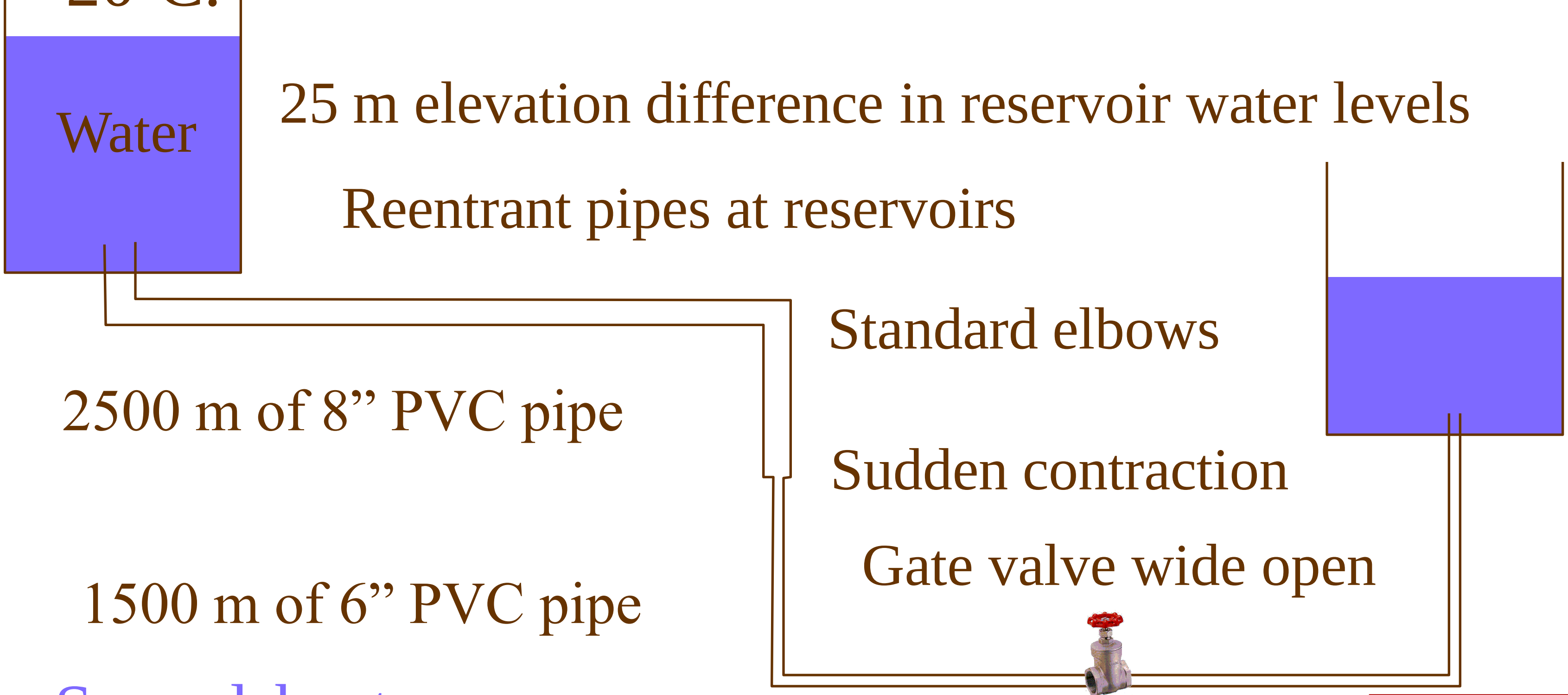
Spreadsheet

# Find D or Q
# Solution Technique 3: assume f

- The friction factor doesn't vary greatly
- If Q is known assume f is 0.02, if D is known assume rough pipe law $\frac{1}{\sqrt{f}} = 2\log\left(\frac{3.7D}{\varepsilon}\right)$
- Use Darcy Weisbach and minor loss equations
- Solve for Q or D
- Calculate Re and ε/D
- Find new f on Moody diagram
- Iterate

# Example: Minor and Major Losses

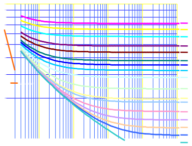
- Find the maximum dependable flow between the reservoirs for a water temperature range of 4ºC to 20ºC.




Spreadsheet

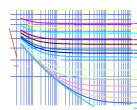
Directions

# Example (Continued)

- What are the Reynolds numbers in the two pipes? 90,000 & 125,000 ε/D= 0.0006, 0.0008
- Where are we on the Moody Diagram? 
- What is the effect of temperature?
- Why is the effect of temperature so small?
- What value of K would the valve have to produce to reduce the discharge by 50%? 140

Spreadsheet

# Example (Continued)

- Were the minor losses negligible? Yes
- Accuracy of head loss calculations? 5%
- What happens if the roughness increases by a factor of 10?  f goes from 0.02 to 0.035
- If you needed to increase the flow by 30% what could you do? Increase small pipe diameter

# Pipe Flow Summary (1)

- Shear increases __linearly__ with distance from the center of the pipe (for both laminar and turbulent flow)
- Laminar flow losses and velocity distributions can be derived based on momentum (Navier Stokes) and energy conservation
- Turbulent flow losses and velocity distributions require _experimental results

# Pipe Flow Summary (2)

- Energy equation left us with the elusive head loss term
- Dimensional analysis gave us the form of the head loss term (pressure coefficient)
- Experiments gave us the relationship between the pressure coefficient and the geometric parameters and the Reynolds number (results summarized on Moody diagram)

# Pipe Flow Summary (3)

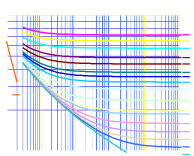
- Dimensionally correct equations fit to the empirical results can be incorporated into computer or calculator solution techniques
- Minor losses are obtained from the pressure coefficient based on the fact that the pressure coefficient is ___constant___ at high Reynolds numbers
- Solutions for discharge or pipe diameter often require iterative or computer solutions

# Directions

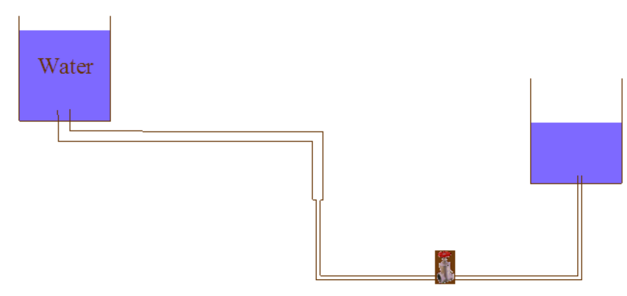
- Assume fully turbulent (rough pipe law)
  - find f from Moody (or from von Karman)
- Find total head loss (draw control volume)
- Solve for Q using symbols (must include minor losses) (no iteration required)

$$h_l = \sum h_f + \sum h_{minor}$$

Solution



Pipe roughness



# Find Q given pipe system

$$h_{minor} = K\frac{8Q^2}{g\pi^2 D^4}$$

$$h_f = \mathrm{f}\frac{8}{g\pi^2}\frac{LQ^2}{D^5}$$

$$h_l = \sum h_{\mathrm{f}} + \sum h_{minor}$$

$$h_l = \frac{8Q^2}{g\pi^2}\left[\sum\left(\mathrm{f}\frac{L}{D^5}\right) + \sum\left(\frac{K}{D^4}\right)\right]$$

$$Q = \pi\sqrt{\frac{gh_l}{8\left[\sum\left(\mathrm{f}\frac{L}{D^5}\right) + \sum\left(\frac{K}{D^4}\right)\right]}}$$