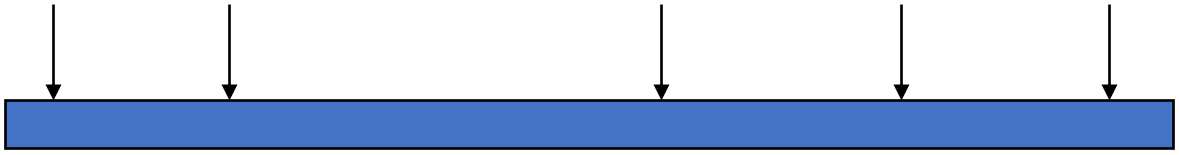
JYOTHISHMATHI INSTITUTE OF TECHNOLOGY & SCIENCE
Nustulapur, Karimnagar - 505481
(Approved by AICTE, New Delhi & Affiliated to JNTUH)

MECHANICS OF SOLIDS

STRESS AND STRAIN IN BEAMS



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II BTECH I SEM 2018-19

BENDING BEAMS

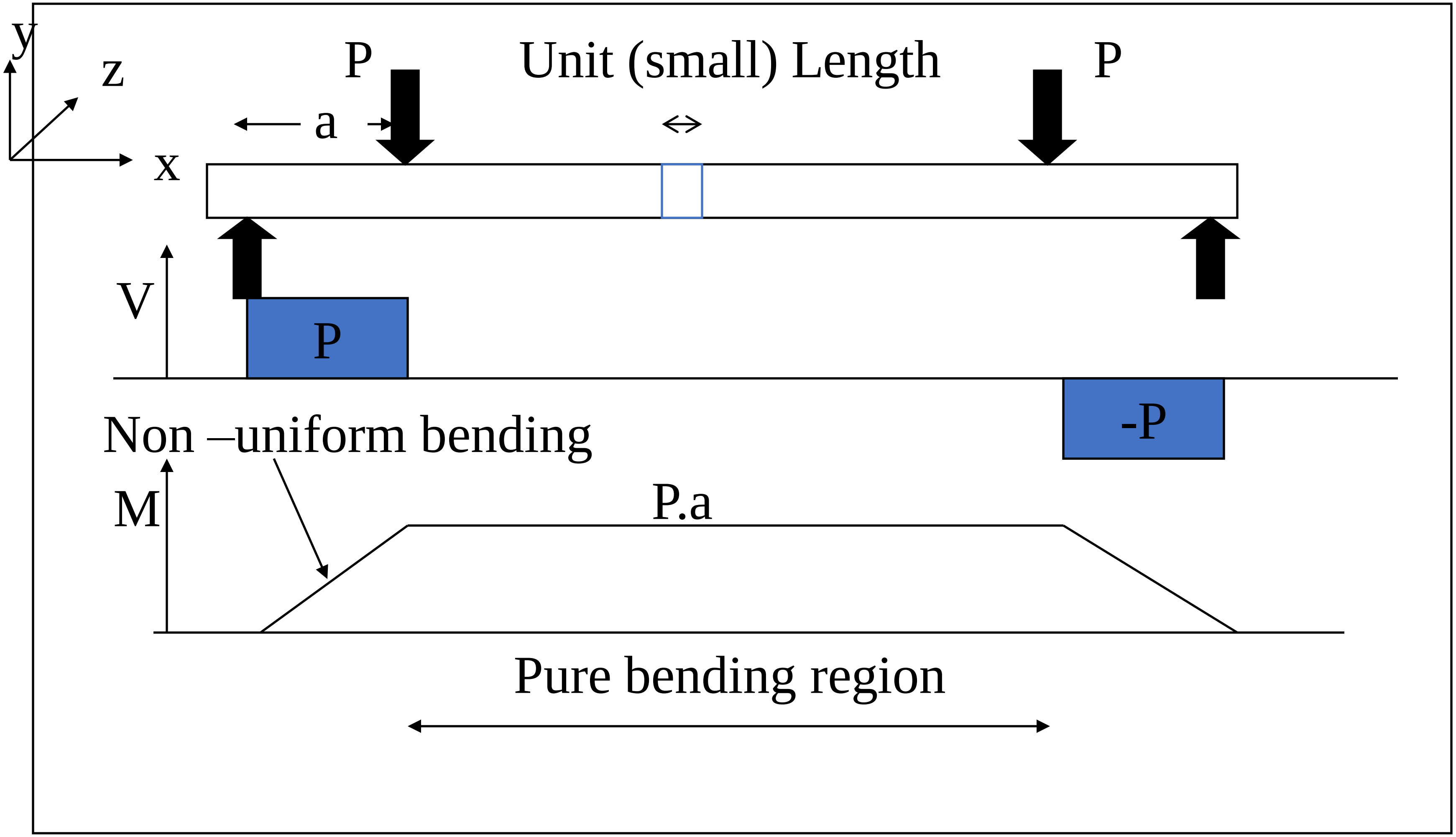
- LOADS ON BEAM PRODUCE STRESS RESULTANTS, V & M

- V & M PRODUCE NORMAL STRESSES AND STRAINS IN PURE BENDING
- V & M PRODUCE ADDITIONAL SHEAR STRESSES IN NON-UNIFORM BENDING

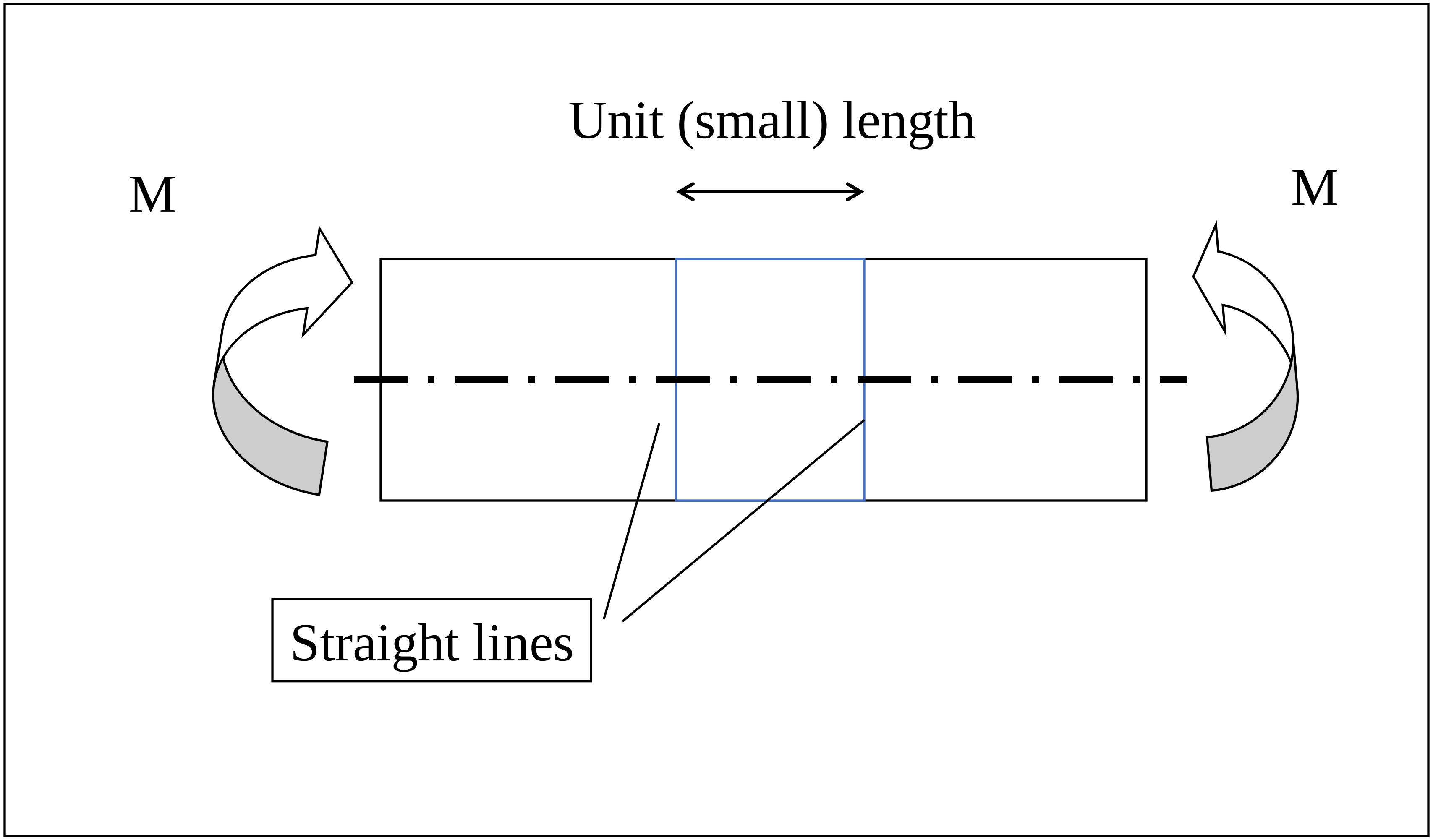
TYPES OF BENDING

- PURE BENDING – FLEXURE UNDER CONSTANT M. i.e. $V = 0 = dM/dx$
- NON-UNIFORM BENDING – FLEXURE WHEN V NON-ZERO

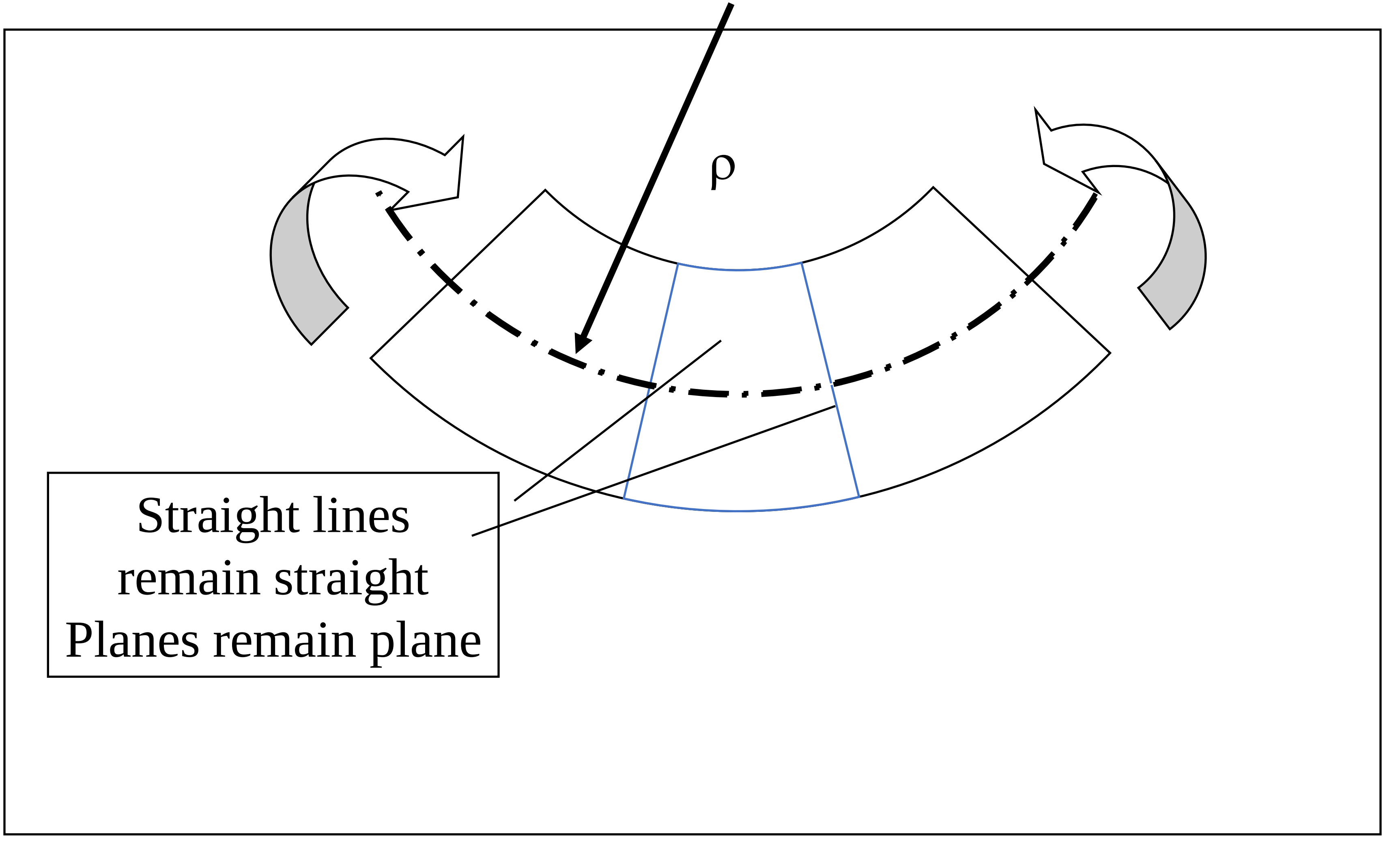
PURE BENDING



PURE BENDING



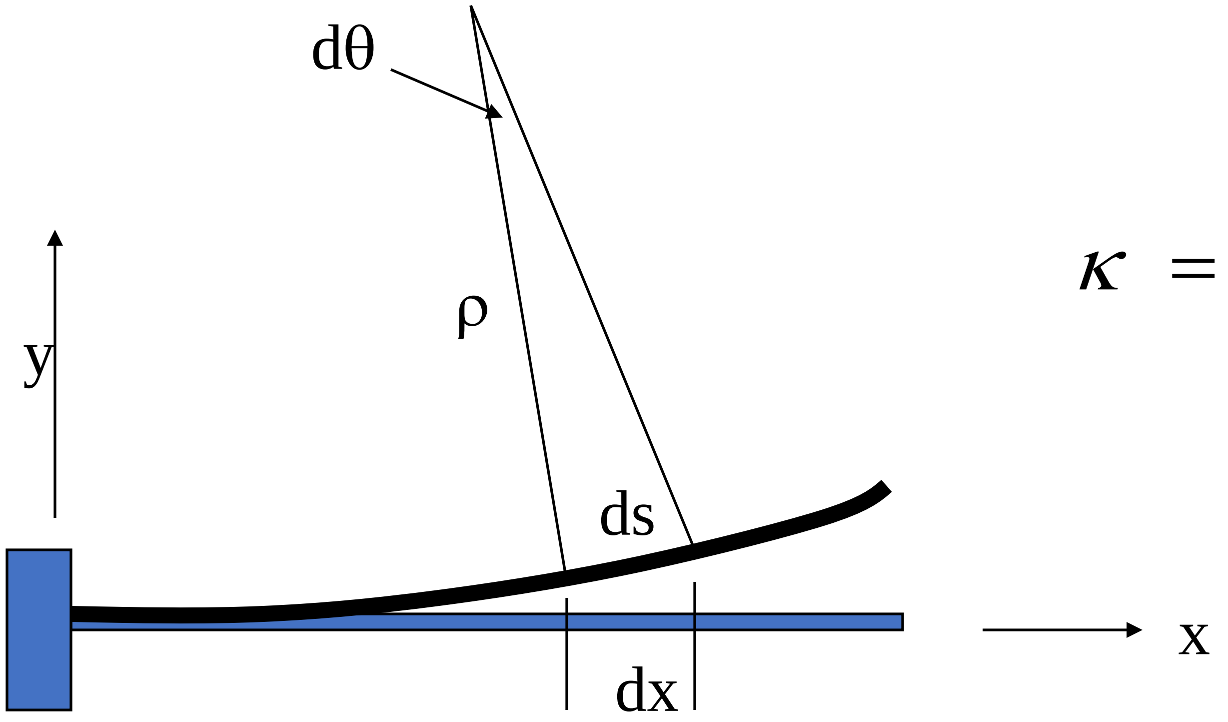
RADIUS OF CURVATURE ρ



CURVATURE $\kappa = 1/\rho$

$$\rho . d\theta = ds = dx$$

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{dx}$$



$$\rho d\theta = dx$$

$$L_1 = (\rho - y)d\theta$$

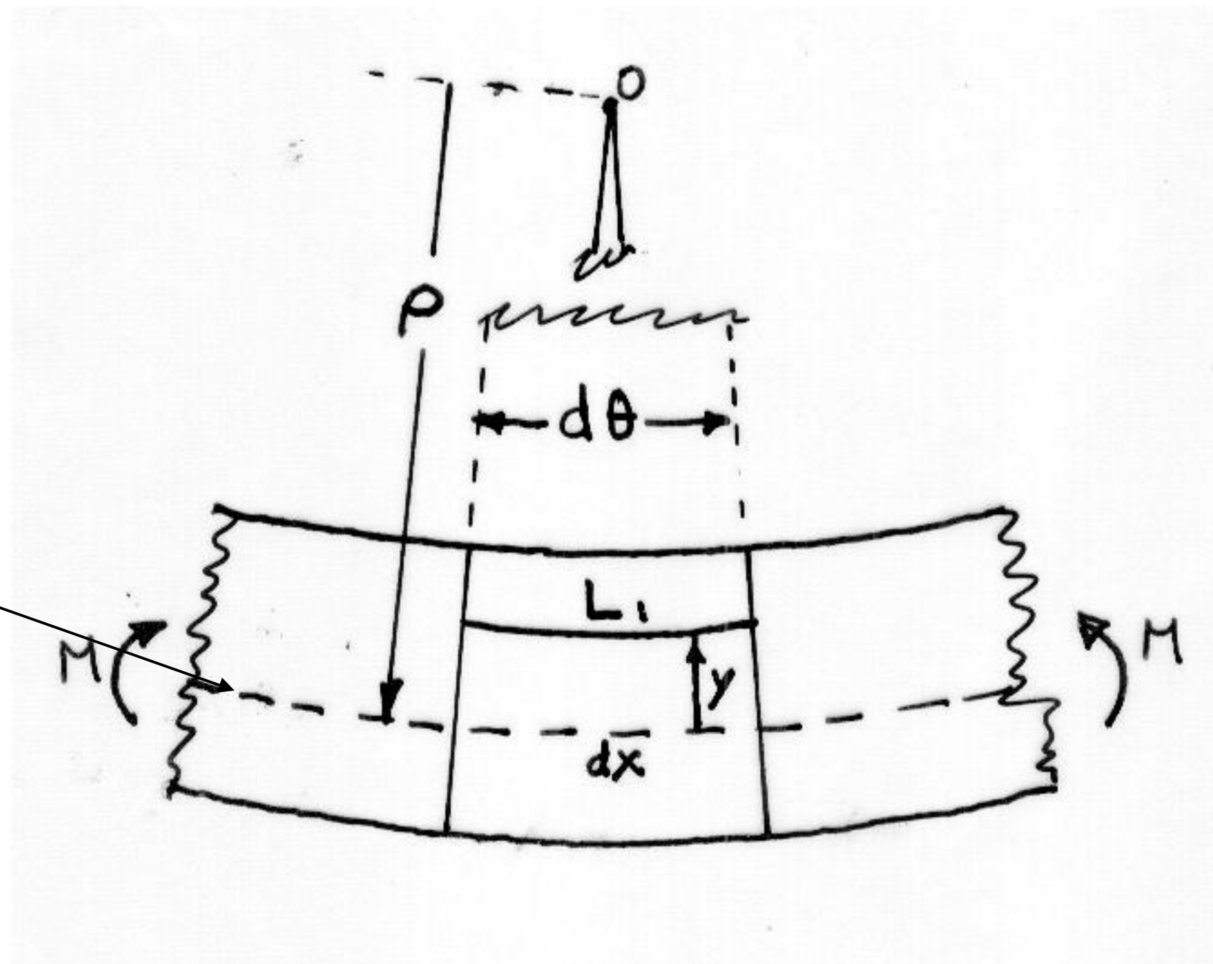
$$L_1 = dx - (y/\rho)dx$$

$$L_1 - dx = -(y/\rho)dx$$

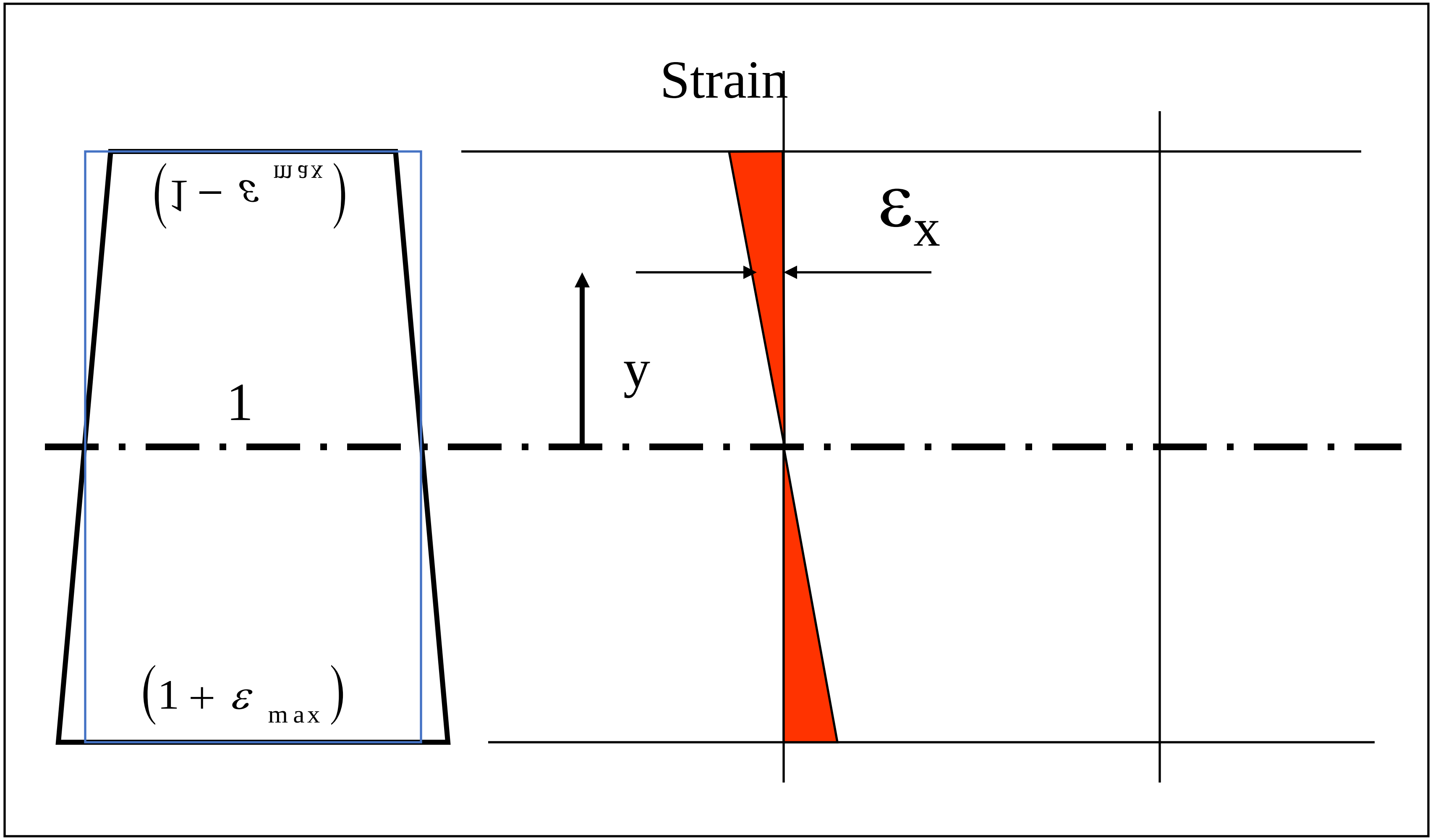
NORMAL STRAIN

$$\varepsilon_x = -(y/\rho)dx/dx = -\kappa . y$$

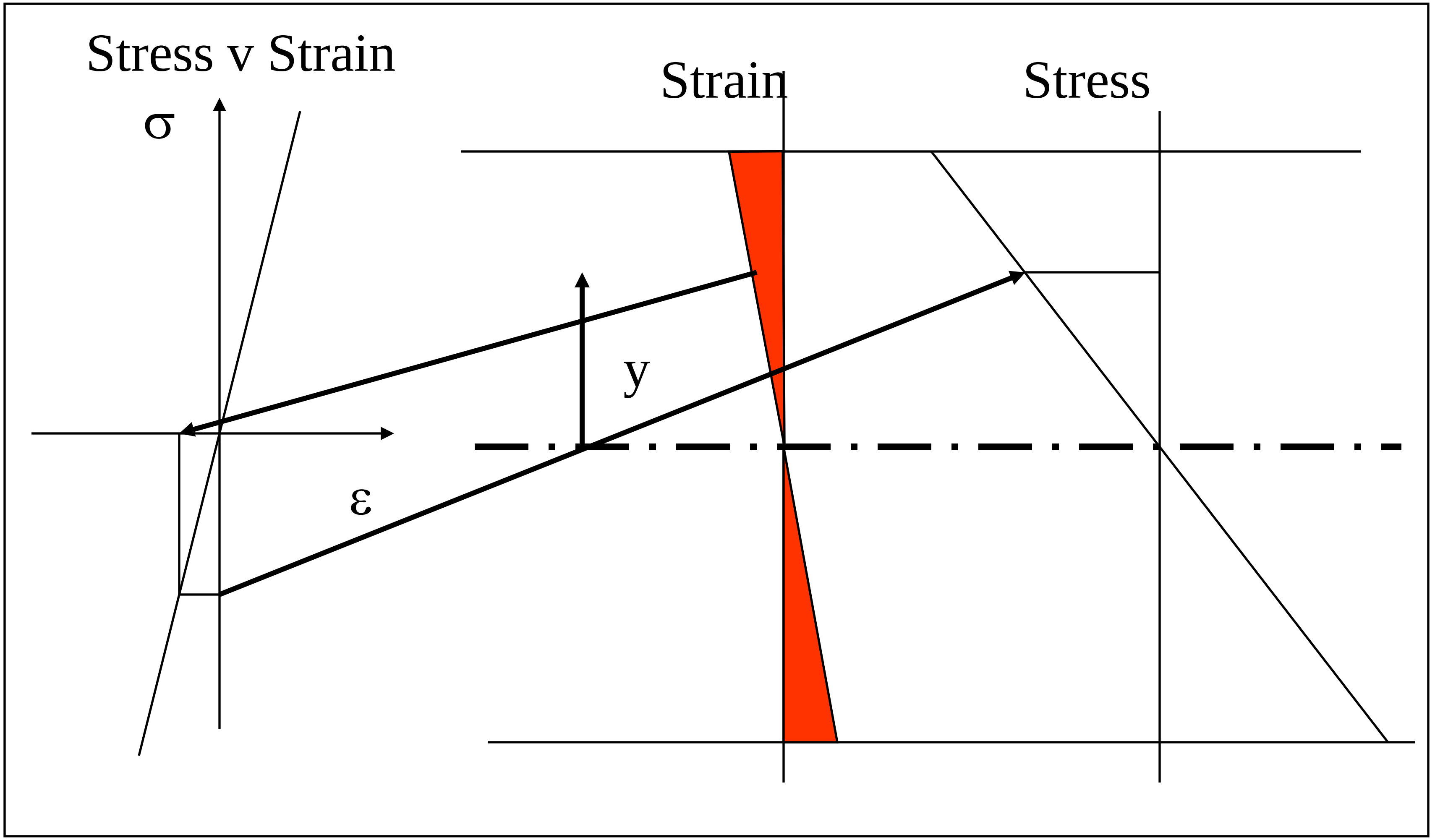
NEUTRAL AXIS



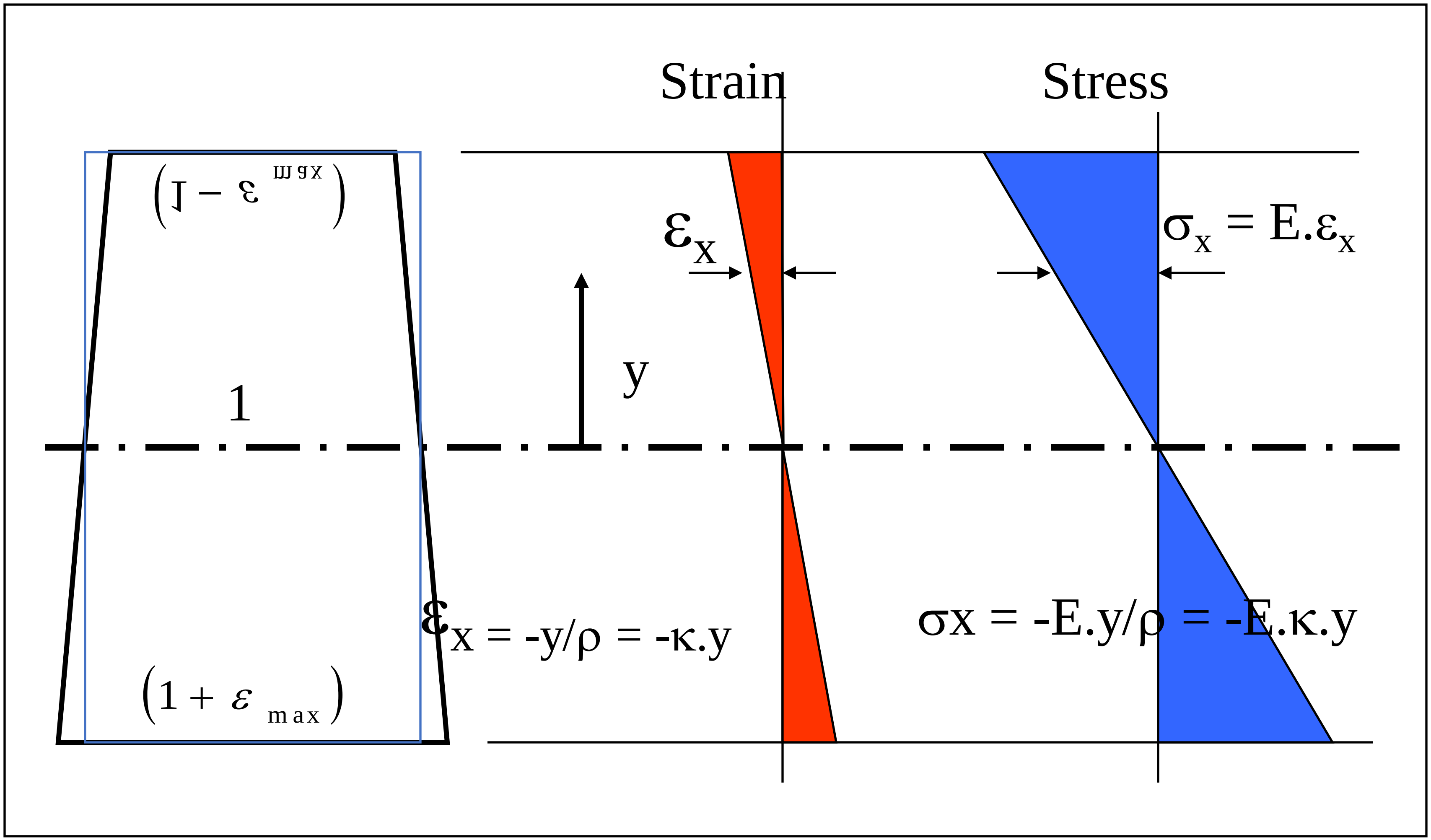
LONGITUDINAL STRAIN



LONGITUDINAL STRESS



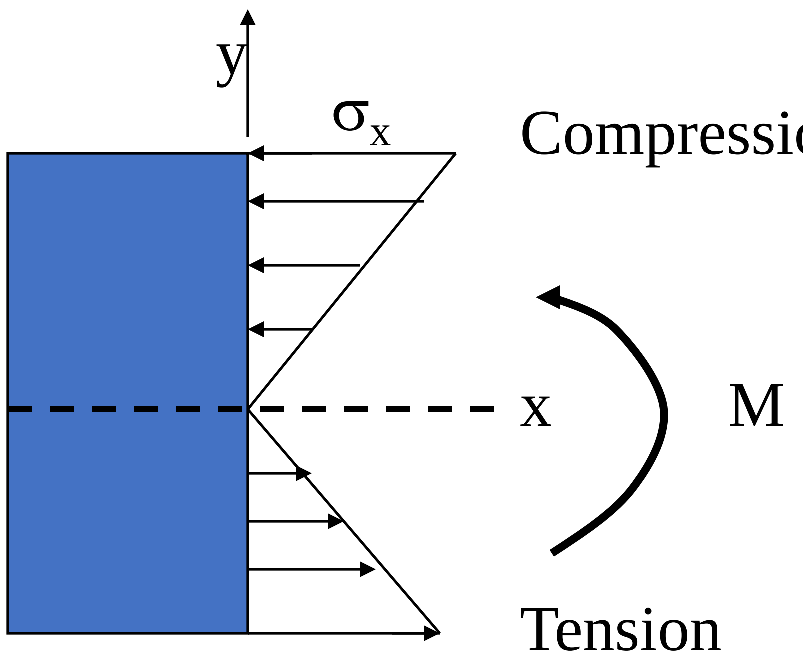
NORMAL STRESS AND STRAIN

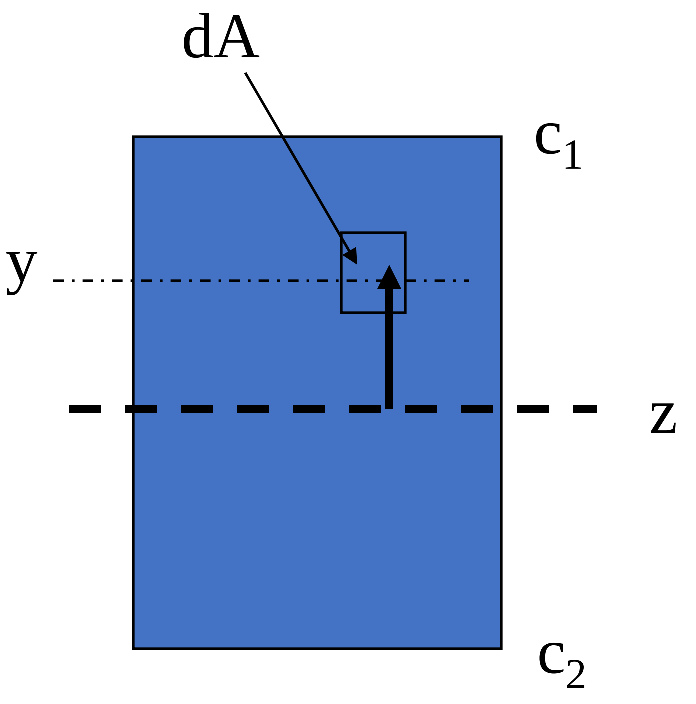


RELATION BETWEEN σ & M

- NEED TO KNOW WHERE NEUTRAL AXIS IS. FIND USING HORIZONTAL FORCE BALANCE
- NEED A RELATION BETWEEN κ & M. FIND USING MOMENT BALANCE –gives THE MOMENT CURVATURE EQUATION & THE FLEXURE FORMULA

NEUTRAL AXIS PASSES THROUGH CENTROID





$$\int_A \sigma_x \, dA = -\int_A E.\kappa.y.dA = 0$$

$$\int_A y.dA = 0$$

First moment of area

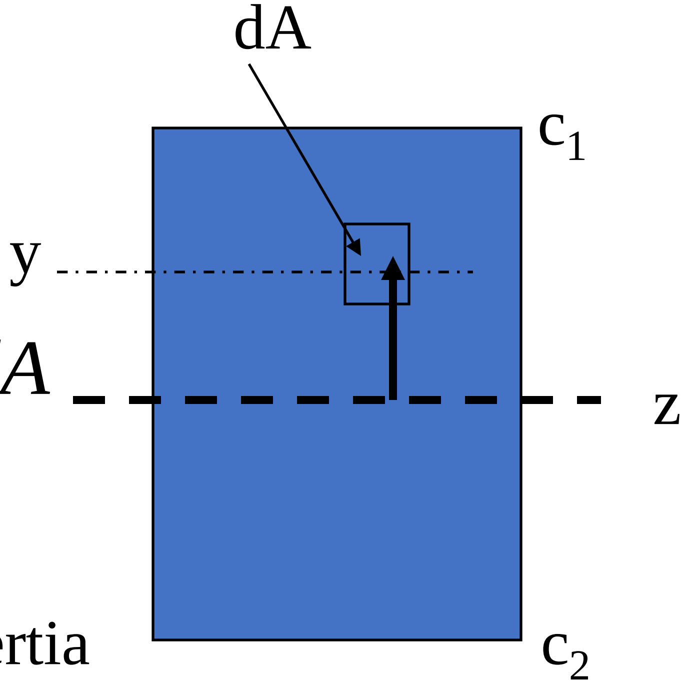
MOMENT-CURVATURE EQN

$$dM = -\sigma_x y dA$$

$$M = -\int_A \sigma_x y dA = \int_A \kappa . E . y^2 dA$$

$$I = \int_A y^2 dA$$

= moment of inertia wrt neutral axis



$$M = \kappa . E . I$$

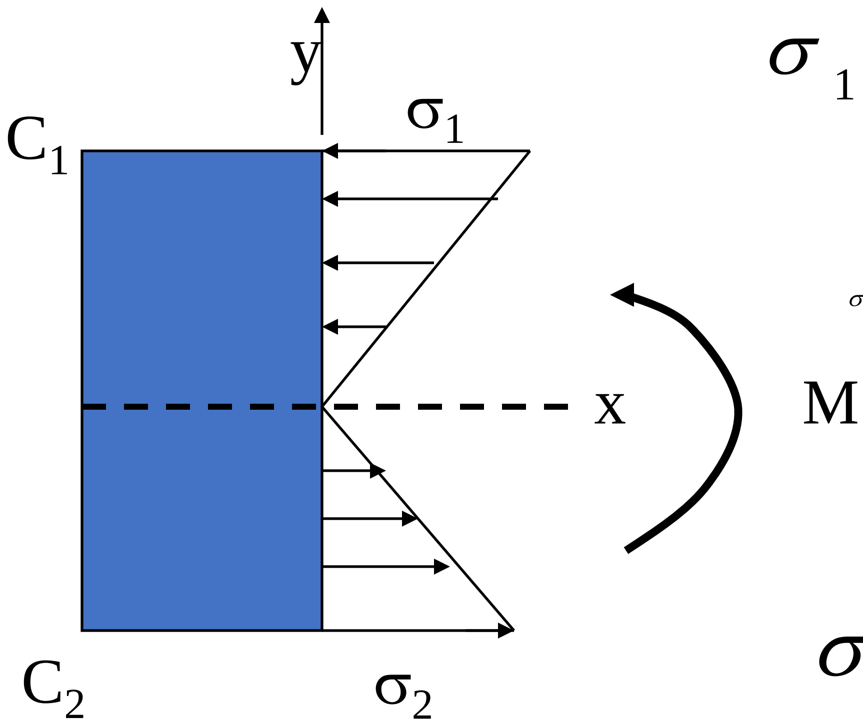
FLEXURE FORMULA FOR BENDING STRESSES

$$\sigma_x = -E.\kappa.y = -\frac{M.y}{I}$$

MAXIMUM STRESSES

OCCUR AT
THE TOP & BOTTOM FACES

MAXIMUM STRESSES



$\sigma 1$

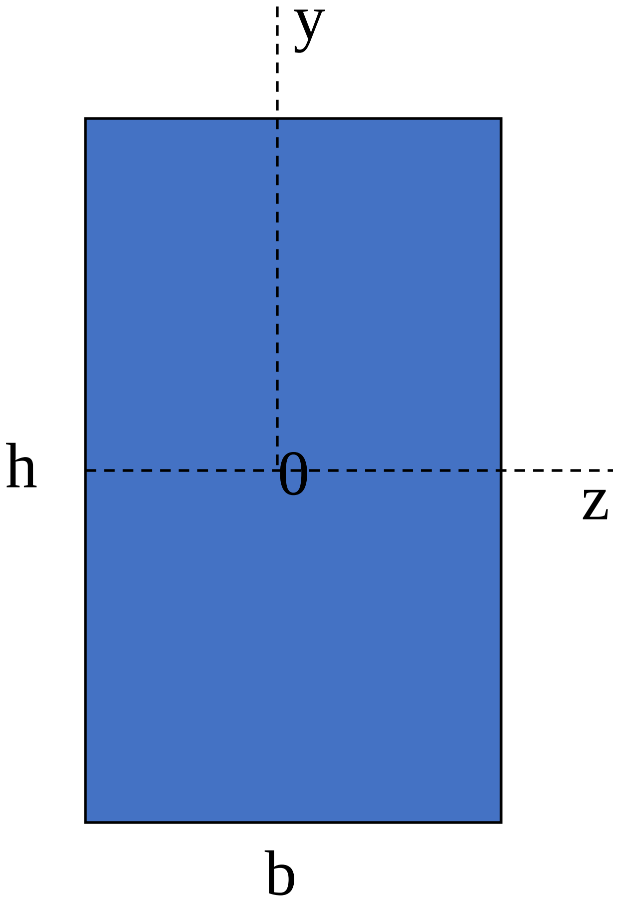
$$\sigma_1 = -\frac{M.c_1}{I} = -\frac{M}{S_1}$$

$$S_1 = I/c_1$$

Section modulus

$$\sigma_2 = -\frac{M.c_2}{I} = -\frac{M}{S_2}$$

I & S FOR BEAM



$$I = \frac{b.h^3}{12}$$

$$S = \frac{b.}{6}h^2$$

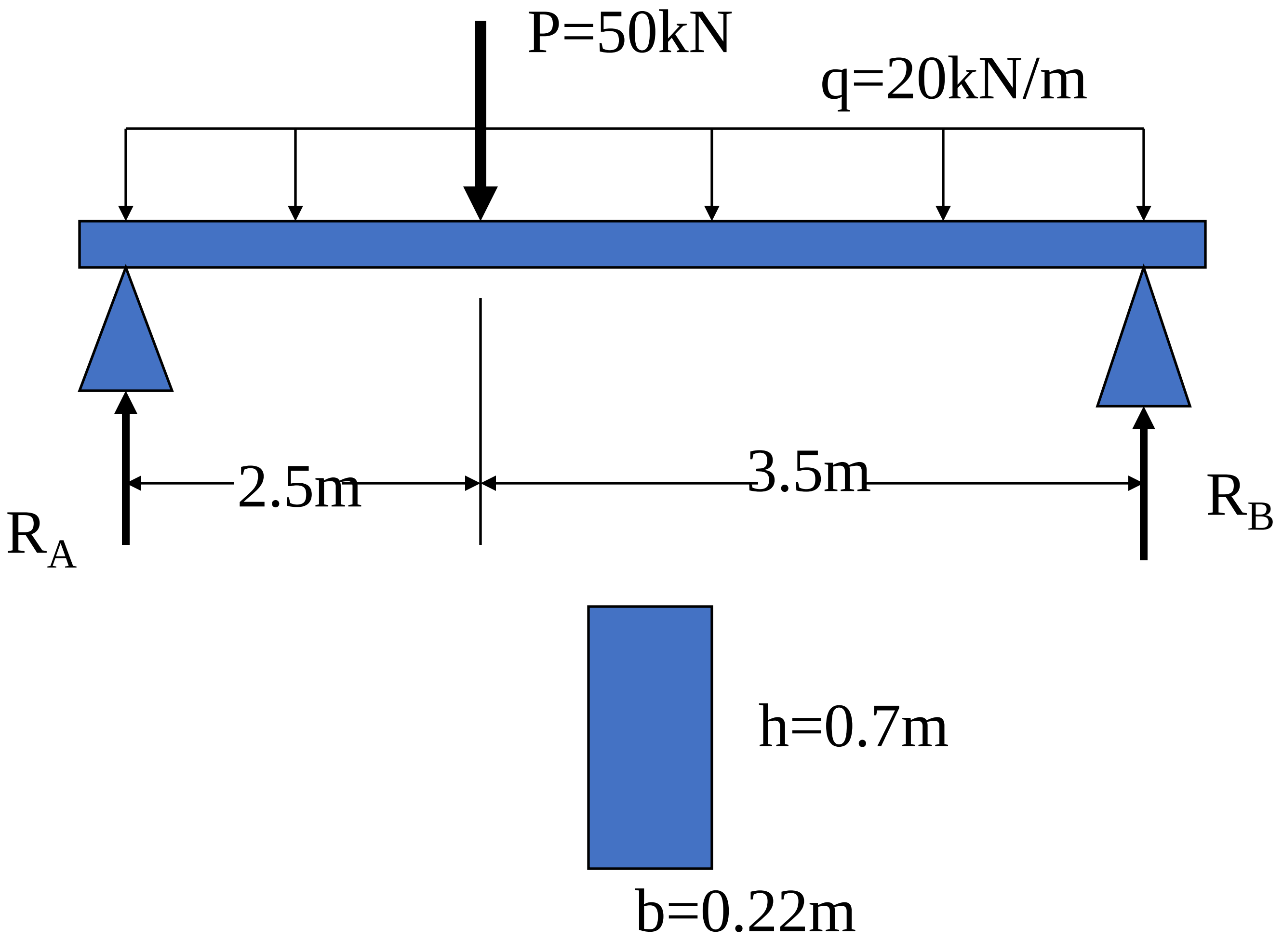
BEAM DESIGN

- SELECT SHAPE AND SIZE SO THAT STRESS DOES NOT EXCEED σ_{ALLOW}
- CALCULATE REQUIRED $S=M_{MAX}/\sigma_{ALLOW}$
- CHOOSE LOWEST CROSS SECTION WHICH SATISFIES S

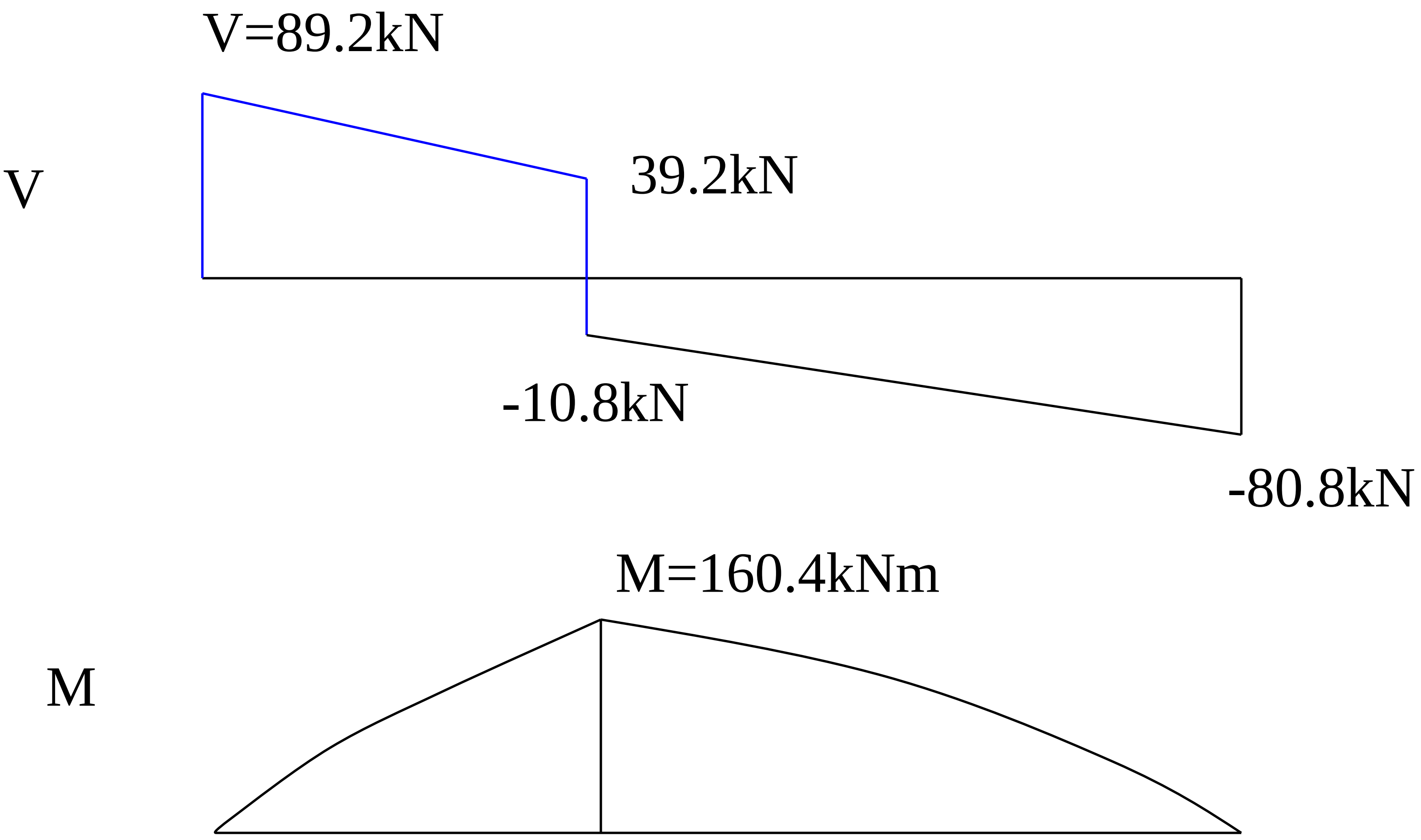
IDEAL BEAM

- RECTANGULAR BEAM, $S_R=bh^2/6=Ah/6$
- CYLINDRICAL BEAM, $S=0.85.S_R$
    - IDEAL BEAM, HALF THE AREA AT h/2
- IDEAL BEAM, $S=3.S_R$
- STANDARD I-BEAM, $S=2.S_R$

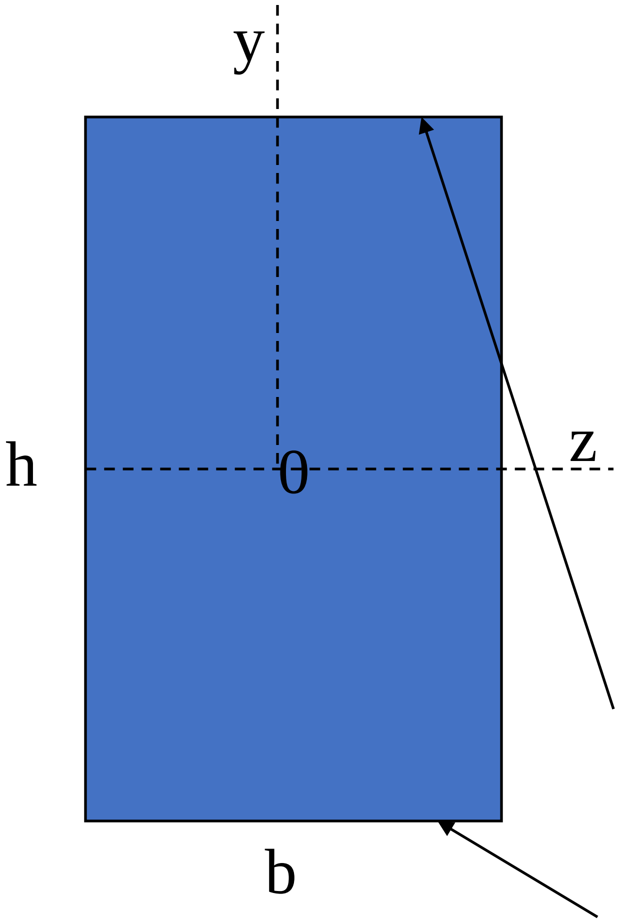
STRESSES CAUSED BY BENDING



V & M DIAGRAMS



σ_{MAX} FOR BEAM



$$S = \frac{b.}{6} h^2 = \frac{0.22 \, x \, 0.7^2}{6} = 0.018 \; m^3$$

MAXIMUM STRESSES

$$\sigma_C = -\frac{M_{\max}}{S} = -\frac{160.4 \, kNm}{0.018 \; m^3} = -8.9 \, MPa$$

$$\sigma_T = +8.9 \, MPa$$