NETWORK THEORY

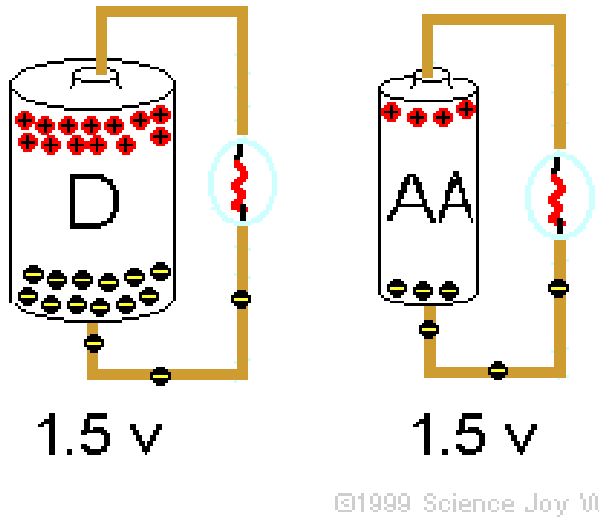
B.BABU

Assistant Professor

Department of EEE

Jyothishmathi Institute of Technology & Science

Potential Difference =Voltage=EMF



In a battery, a series of chemical reactions occur in which electrons are transferred from one terminal to another. There is a **potential difference (voltage)** between these poles.

$$\text{Voltage} = \text{Potential Difference} = \text{Emf}$$

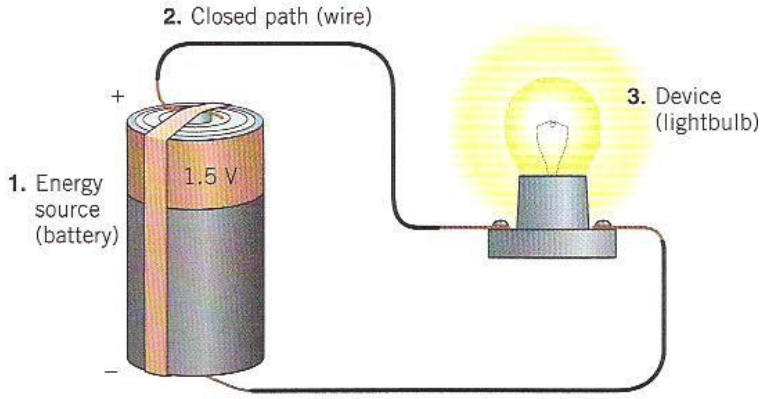
$$V = \Delta V = \varepsilon$$

The **<u>maximum</u> potential difference** a power source can have is called the **electromotive force or (EMF),** ε. The term isn't actually a force, simply the amount of energy per charge (J/C or V)

All electric circuits have three main parts

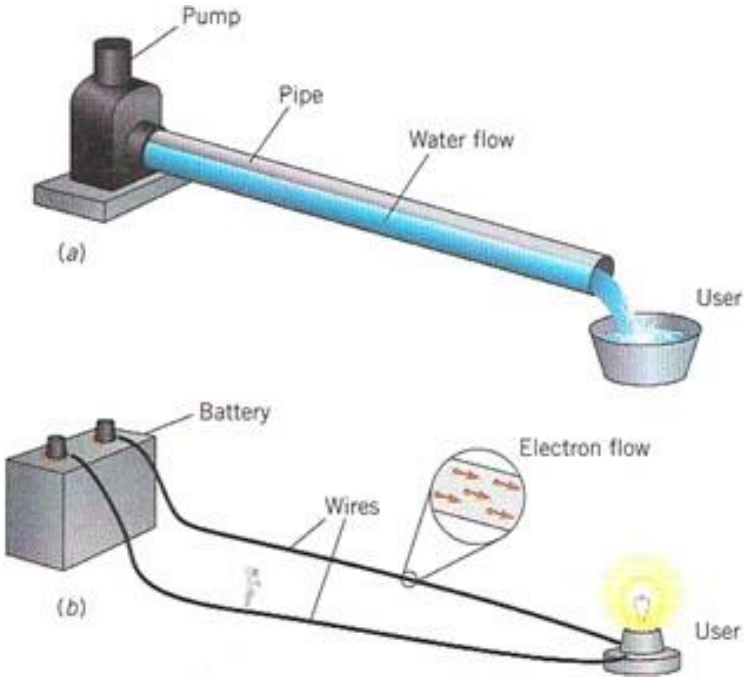
1. A source of energy
2. A closed path
3. A device which uses the energy

If ANY part of the circuit is open the device will not work!



Electricity can be symbolic

Circuits are very similar to water flowing through a pipe



A pump basically works on TWO IMPORTANT PRINCIPLES concerning its flow

- There is a **PRESSURE DIFFERENCE** where the flow begins and ends
- A certain **AMOUNT** of flow passes each **SECOND**.

A circuit basically works on TWO IMPORTANT PRINCIPLES

- There is a "**POTENTIAL DIFFERENCE aka VOLTAGE**" from where the charge begins to where it ends
- The **AMOUNT of CHARGE** that flows **PER SECOND** is called **CURRENT.**

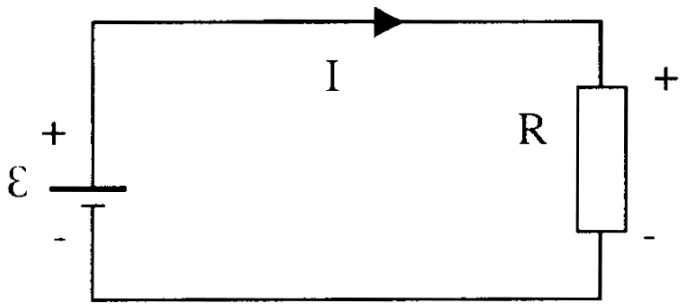
Current

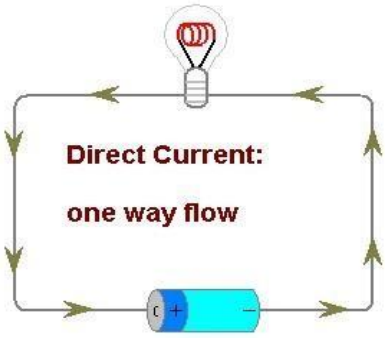
Current is defined as the rate at which charge flows through a surface.

$$I = \frac{q}{t} = \frac{Coloumbs(C)}{Second(s)} = Amperes = Amps = A$$

The current is in the same direction as the flow of positive charge (for this course)

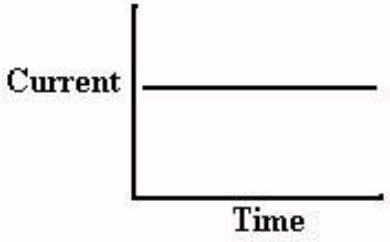
Note: The "I" stands for *intensity*



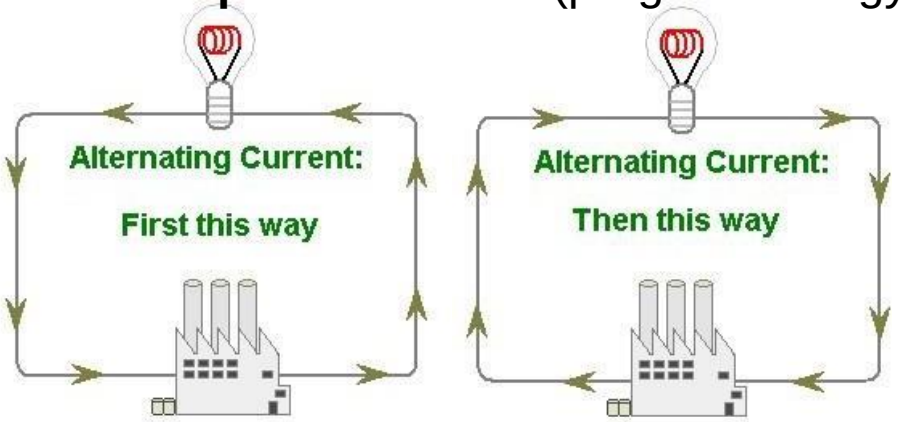


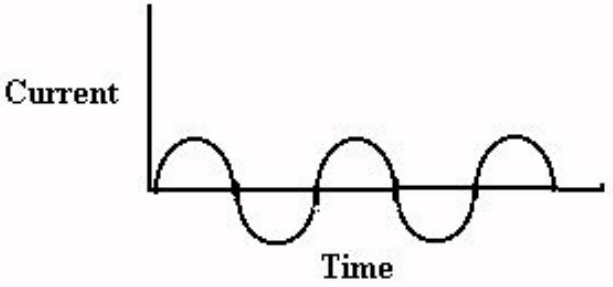
DC = Direct Current - current flows in one direction

Example: Battery



AC = Alternating Current- current reverses direction many times per second. This suggests that AC devices turn OFF and ON. **Example:** Wall outlet (progress energy)





Ohm's Law

"The voltage (potential difference, emf) is directly related to the current, when the resistance is constant"

$\Delta V \alpha I$

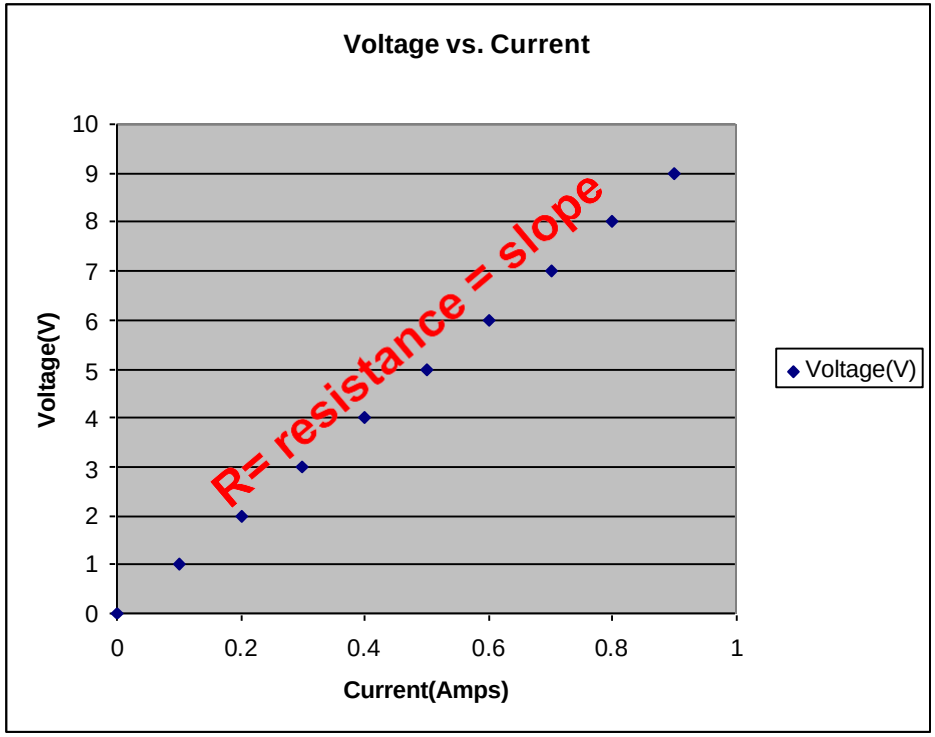
$R =$ constant of proportion ality

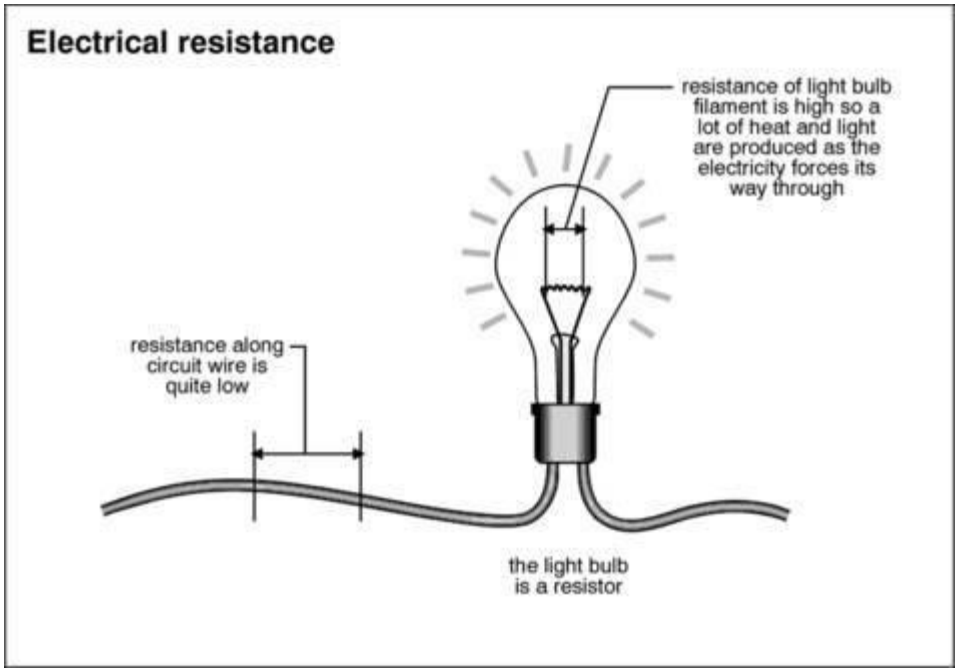
$R =$ Resistance

$$\Delta V = IR$$

$$\varepsilon = IR$$

Since R=ΔV/I, the resistance is the SLOPE of a ΔV vs. I graph

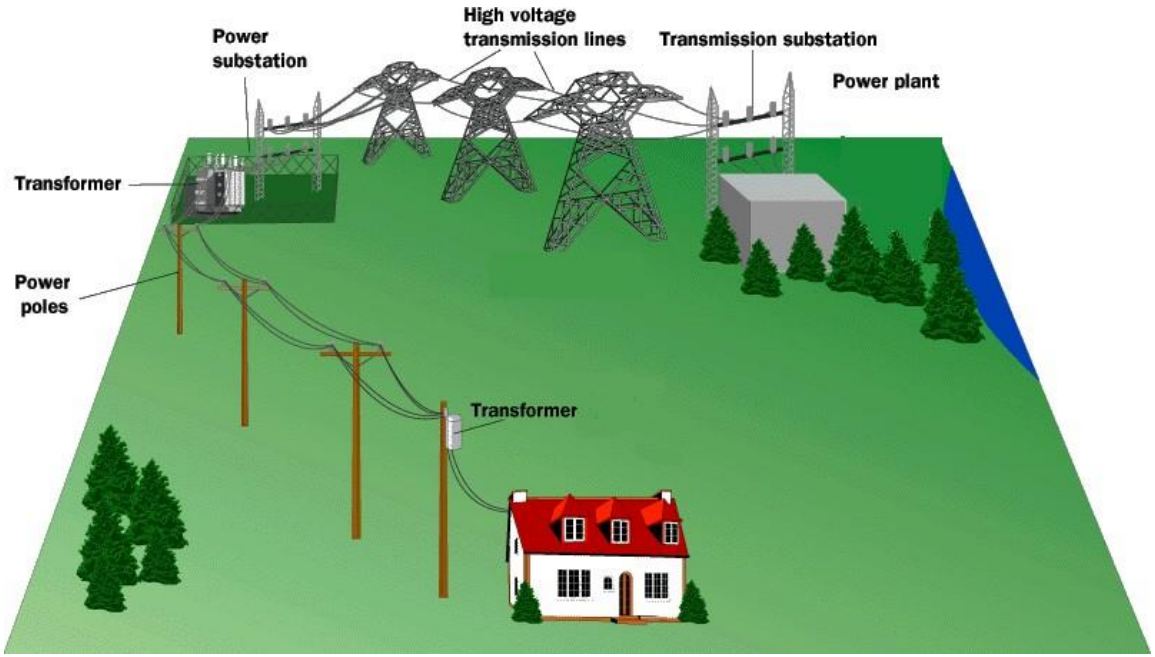




The unit for resistance is the OHM, Ω

Electrical POWER

We have already learned that POWER is the rate at which work (energy) is done. Circuits that are a prime example of this as batteries only last for a certain amount of time AND we get charged an energy bill each month based on the amount of energy we used over the course of a month…aka POWER.



POWER

It is interesting to see how certain electrical variables can be used to get POWER. Let's take Voltage and Current for example.

$$V = \frac{Joules}{Coloumb}$$

$$I = \frac{Coloumbs}{Second}$$

$$VxI = \frac{\textbf{Joules} \bullet \textbf{Coloumbs}}{\textbf{Coloumbs} \bullet \textbf{seconds}} = \frac{Joules}{Second} = WATT!$$

$$Power = P = VI$$

Other useful power formulas

$$P = VI$$

$$V = IR$$

$$P = (IR)I = I^2R$$

$$I = \frac{V}{R}$$

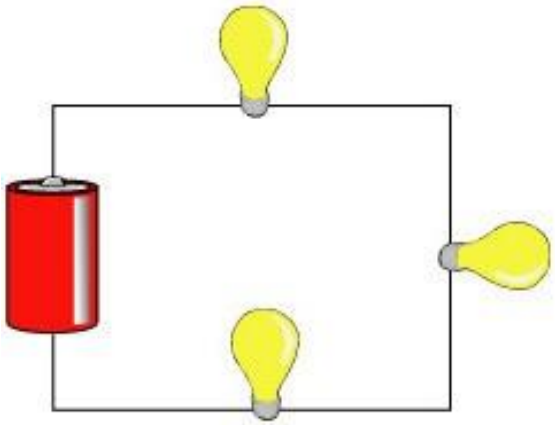
$$P = V\left(\frac{V}{R}\right) = \frac{V^2}{R}$$

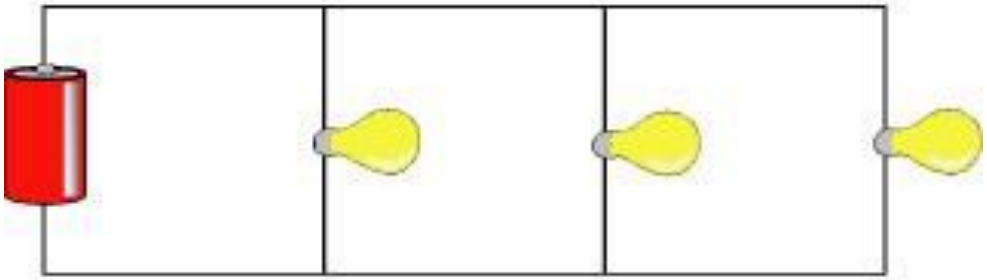
These formulas can also be used! They are simply derivations of the POWER formula with different versions of Ohm's law substituted in.

Ways to Wire Circuits

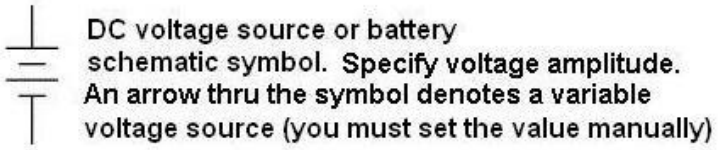
There are 2 basic ways to wire a circuit. Keep in mind that a resistor could be ANYTHING (bulb, toaster, ceramic material…etc)

Series – One after another
Parallel – between a set of junctions and parallel to each other

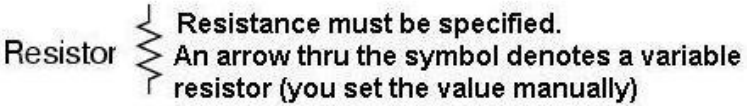




Schematic Symbols



DC voltage source or battery schematic symbol. Specify voltage amplitude. An arrow thru the symbol denotes a variable voltage source (you must set the value manually)

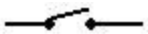


Resistor

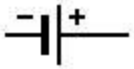
Resistance must be specified. An arrow thru the symbol denotes a variable resistor (you set the value manually)

Voltmeter

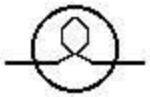
Ammeter



Switch



Battery



Light Bulb

For the battery symbol, the LONG line is considered to be the POSITIVE terminal and the SHORT line , NEGATIVE.

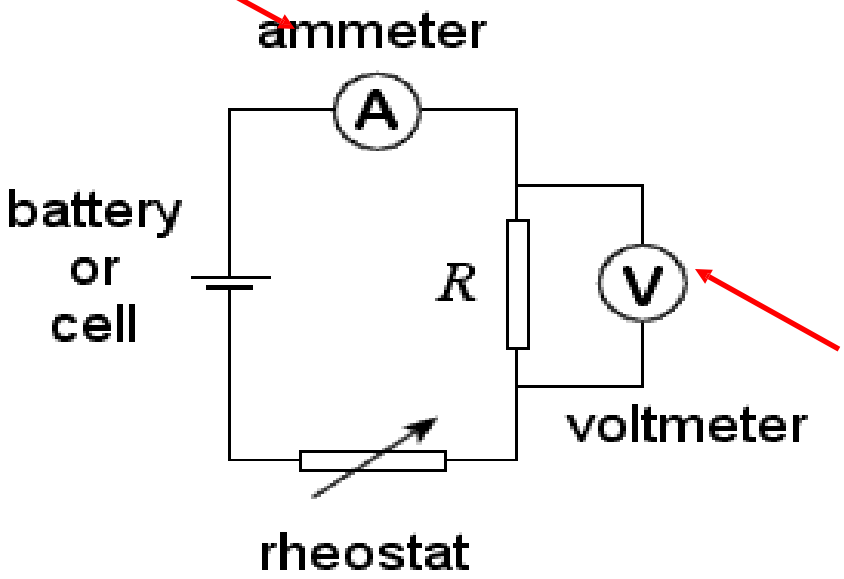
The VOLTMETER and AMMETER are special devices you place IN or AROUND the circuit to measure the VOLTAGE and CURRENT.

The Voltmeter & Ammeter

Current goes THROUGH the ammeter

The voltmeter and ammeter cannot be just placed anywhere in the circuit. They must be used according to their DEFINITION.

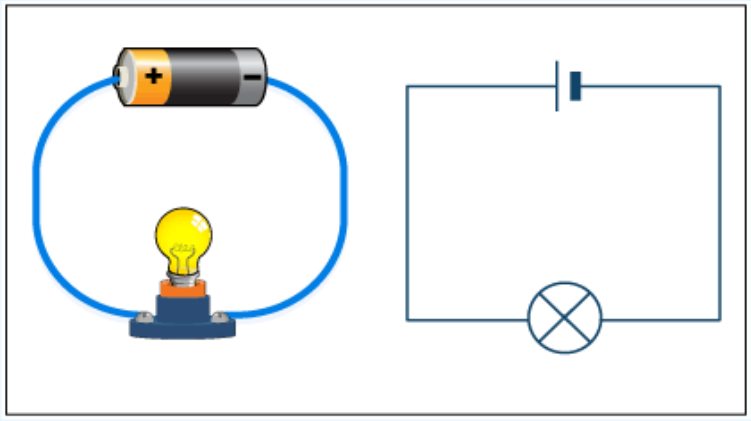
Since a voltmeter measures voltage or POTENTIAL DIFFERENCE it must be placed **ACROSS** the device you want to measure. That way you can measure the CHANGE on either side of the device.

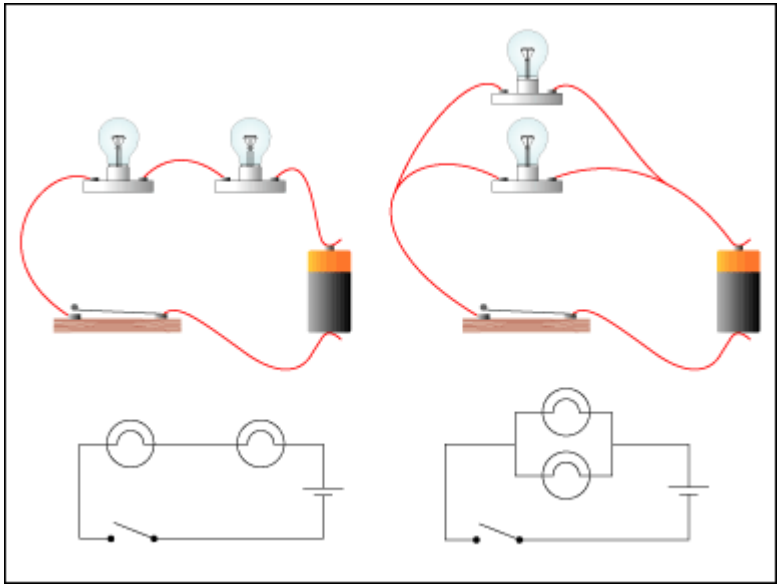


Voltmeter is drawn ACROSS the resistor

Since the ammeter measures the current or FLOW it must be placed in such a way as the charges go **THROUGH** the device.

Simple Circuit

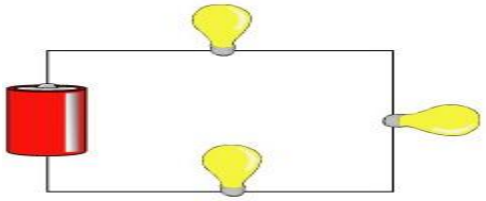




When you are drawing a circuit it may be a wise thing to start by drawing the battery first, then follow along the loop (closed) starting with positive and drawing what you see.

Series Circuit

In in series circuit, the resistors are wired one after another. Since they are all part of the SAME LOOP they each experience the SAME



$$I_{(series)Total} = I_1 = I_2 = I_3$$

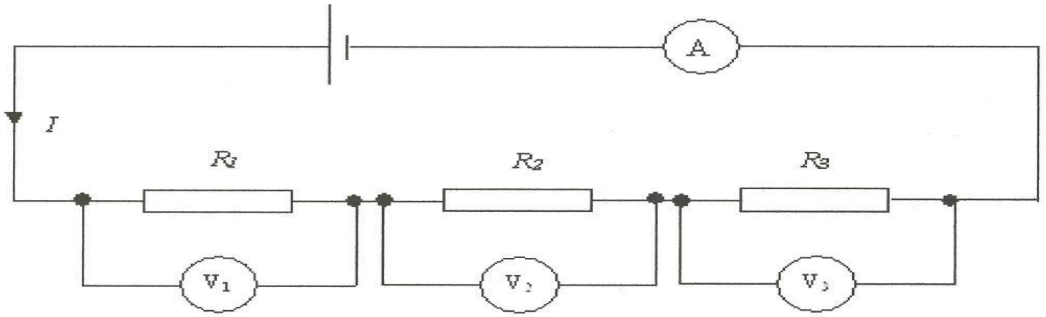
$$V_{(series)Total} = V_1 + V_2 + V_3$$

AMOUNT of current. In figure, however, you see that they all exist BETWEEN the terminals of the battery, meaning they SHARE the potential (voltage).

Series Circuit

$$I_{(series)Total} = I_1 = I_2 = I_3$$

$$V_{(series)Total} = V_1 + V_2 + V_3$$



As the current goes through the circuit, the charges must USE ENERGY to get through the resistor. So each individual resistor will get its own individual potential voltage). We call this VOLTAGE DROP.

$$V_{(series)Total} = V_1 + V_2 + V_3; \quad \Delta V = IR$$

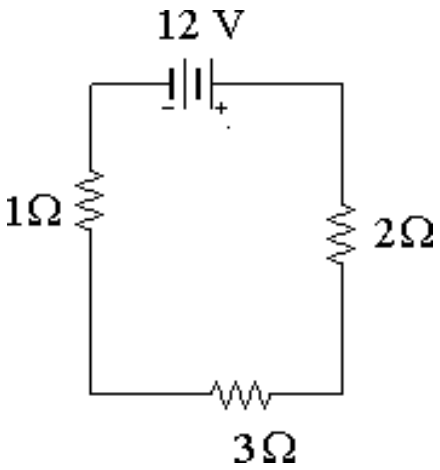
$$(I_T R_T)_{series} = I_1 R_1 + I_2 R_2 + I_3 R_3$$

$$R_{series} = R_1 + R_2 + R_3$$

$$R_s = \sum R_i$$

Note: They may use the terms —effective‖ or —equivalent‖ to mean TOTAL!

Example



A series circuit is shown to the left.

a) What is the total resistance?

R(series) = 1 + 2 + 3 = 6Ω

b) What is the total current?

ΔV=IR 12=I(6) I = 2A

c) What is the current across EACH resistor?

They EACH get 2 amps!

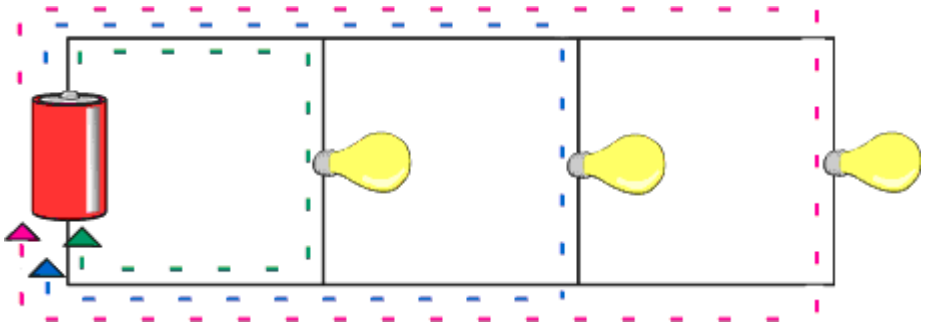
d) What is the voltage drop across each resistor?(Apply Ohm's law to each resistor separately)

$V_{1\Omega}=(2)(1)=$ 2 V $V_{3\Omega}=(2)(3)=$ 6V $V_{2\Omega}=(2)(2)=$ 4V

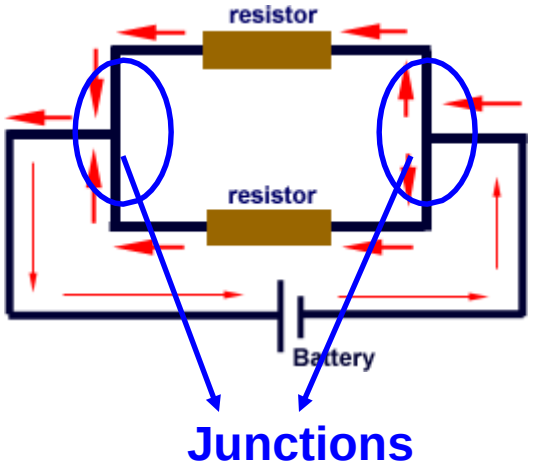
Notice that the individual VOLTAGE DROPS add up to the TOTAL!!

Parallel Circuit

In a parallel circuit, we have multiple loops. So the current splits up among the loops with the individual loop currents **adding** to the total current



It is important to understand that parallel circuits will all have some position where the current splits and comes back together. We call these **JUNCTIONS**.



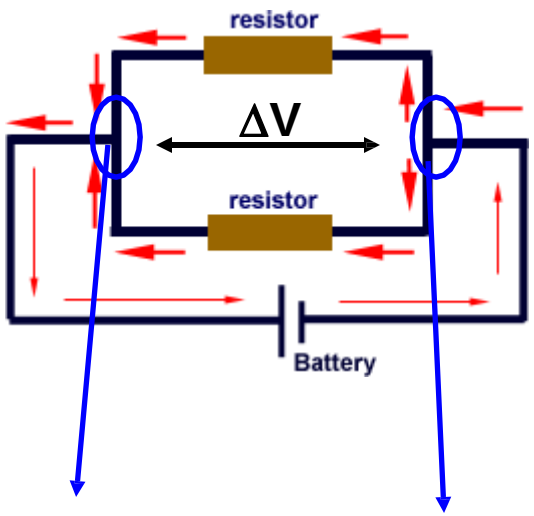
$$I_{(parallel)Total} = I_1 + I_2 + I_3$$

Regarding Junctions :

$$I_{IN} = I_{OUT}$$

The current going IN to a junction will always equal the current going OUT of a junction.

Parallel Circuit



This junction touches the **POSITIVE** terminal

This junction touches the **NEGATIVE** terminal

Notice that the JUNCTIONS both touch the POSTIVE and NEGATIVE terminals of the battery. That means you have the SAME potential difference down EACH individual branch of the parallel circuit. This means that the individual voltages drops are equal.

$$V_{(parallel)Total} = V_1 = V_2 = V_3$$

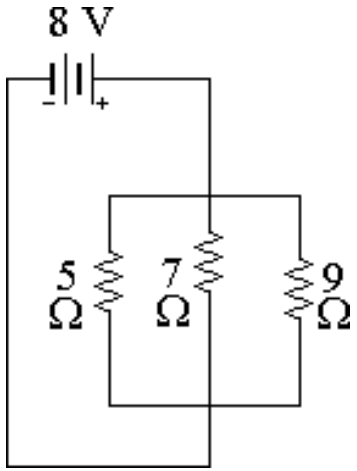
$$I_{(parallel)Total} = I_1 + I_2 + I_3; \Delta V = IR$$

$$\left(\frac{V}{R}\right)_{Parallel} = \frac{V_T}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_P} = \sum \frac{1}{R_i}$$

Example



To the left is an example of a parallel circuit.

a) What is the total resistance?

$$\frac{1}{R_P} = \frac{1}{5} + \frac{1}{7} + \frac{1}{9}$$

$$\frac{1}{R_p} = 0.454 \rightarrow R_P = \frac{1}{0.454} = \textbf{2.20 Ω}$$

b) What is the total current?

$$\Delta V = IR$$

$$8 = I(R) = \textbf{3.64 A}$$

8 V each!

c) What is the voltage across EACH resistor?

d) What is the current drop across each resistor? (Apply Ohm's law to each resistor separately)

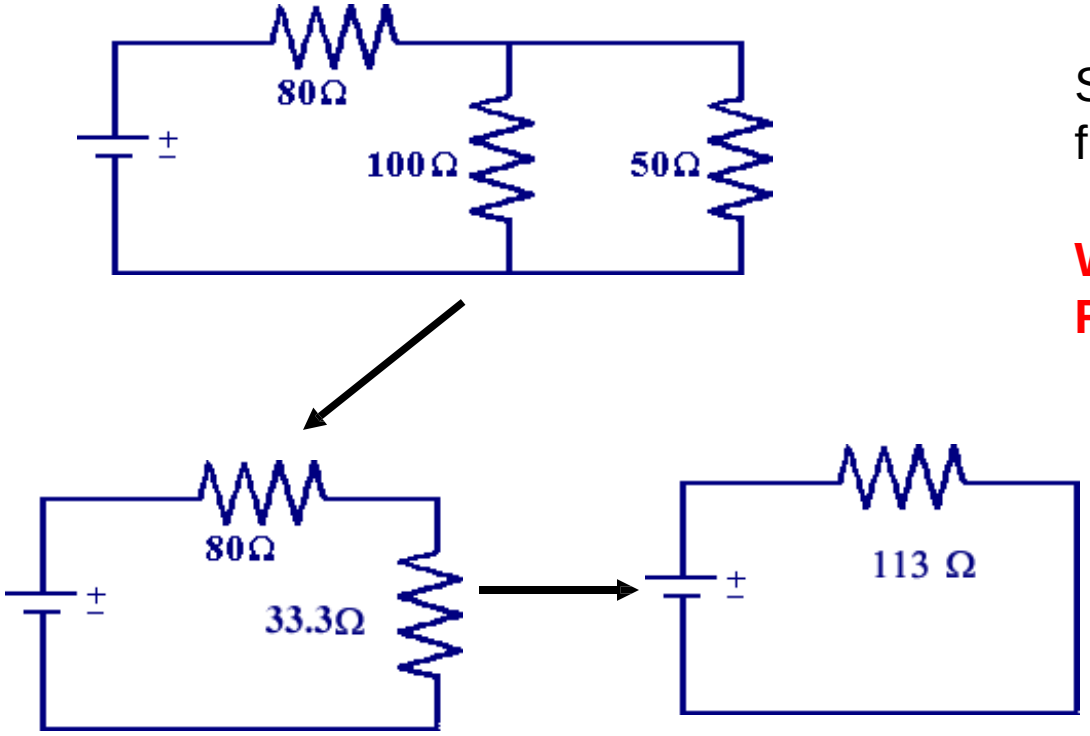
$$\Delta V = IR$$

$$I_{5\Omega} = \frac{8}{5} = \textbf{1.6 A} \qquad I_{7\Omega} = \frac{8}{7} = \textbf{1.14 A} \qquad I_{9\Omega} = \frac{8}{9} = \textbf{0.90 A}$$

Notice that the individual currents ADD to the total.

Compound (Complex) Circuits

Many times you will have series and parallel in the SAME circuit.



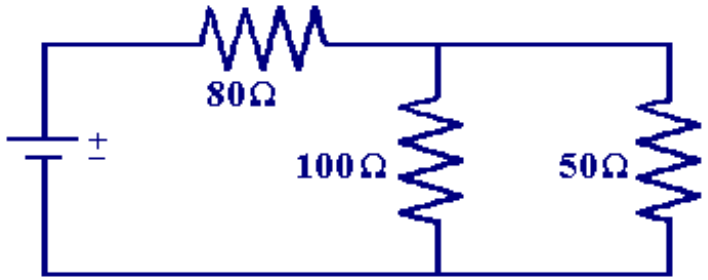
Solve this type of circuit from the inside out.

WHAT IS THE TOTAL RESISTANCE?

$$\frac{1}{R_P} = \frac{1}{100} + \frac{1}{50}; \quad R_P = 33.3\ \Omega$$

$$R_s = 80 + 33.3 = 113.3\ \Omega$$

Compound (Complex) Circuits



$$\frac{1}{R_P} = \frac{1}{100} + \frac{1}{50}; \quad R_P = 33.3\ \Omega$$

$$R_s = 80 + 33.3 = 113.3\ \Omega$$

Suppose the potential difference (voltage) is equal to **120V**. What is the total current?

$$\Delta V_T = I_T R_T$$

$$120 = I_T (113.3)$$

$$I_T = \textbf{1.06 A}$$

What is the VOLTAGE DROP across the 80Ω resistor?

$$\Delta V_{80\,\Omega} = I_{80\,\Omega} R_{80\,\Omega}$$

$$V_{80\,\Omega} = (1.06)(80)$$

$$V_{80\,\Omega} = \textbf{84.8 V}$$

Compound (Complex) Circuits

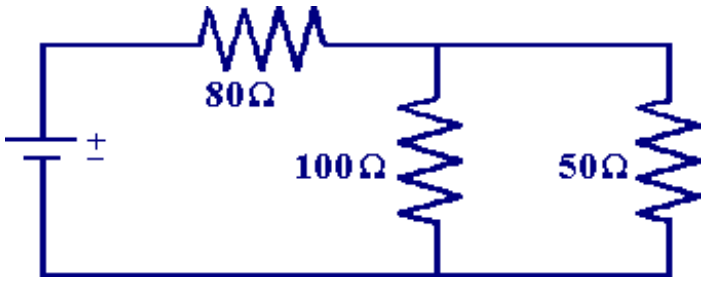
$R_T = 113.3\Omega$

$V_T = 120\ V$

$I_T = 1.06\ A$

$V_{80\Omega} = 84.8V$

$I_{80\ \Omega} = 1.06\ A$



What is the VOLTAGE DROP across the 100Ω and 50Ω resistor?

$V_{T\ (parallel)} = V_2 = V_3$

$V_{T\ (series)} = V_1 + V_{2\&3}$

$120 = 84.8 + V_{2\&3}$

$V_{2\&3} =$ **35.2 V Each!**

What is the current across the 100Ω and 50Ω resistor?

$I_{T\ (parallel)} = I_2 + I_3$

$I_{T\ (series)} = I_1 = I_{2\&3}$

$I_{100\ \Omega} = \frac{35.2}{100} =$ **0.352 A**

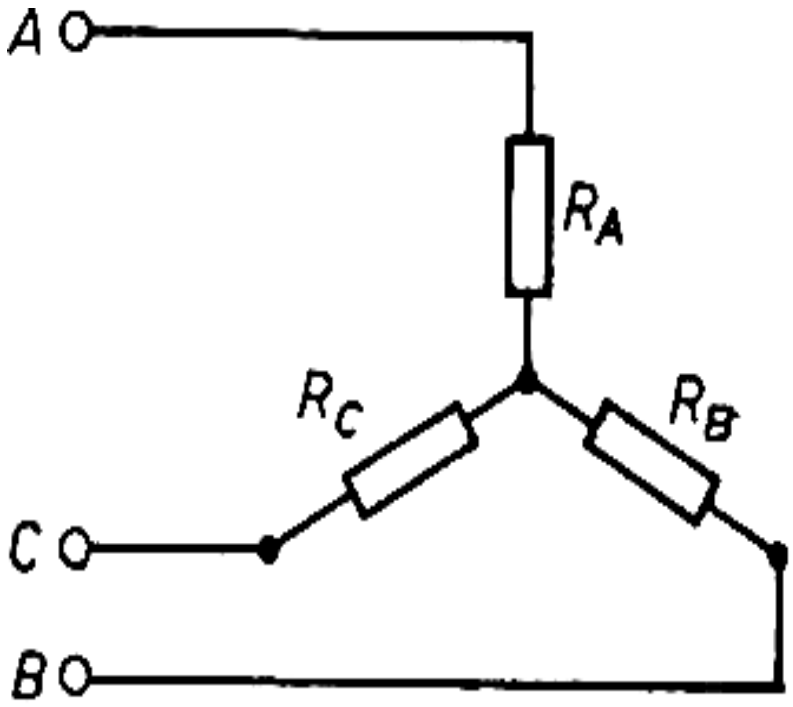
$I_{50\ \Omega} = \frac{35.2}{50} =$ **0.704 A**

Add to 1.06A

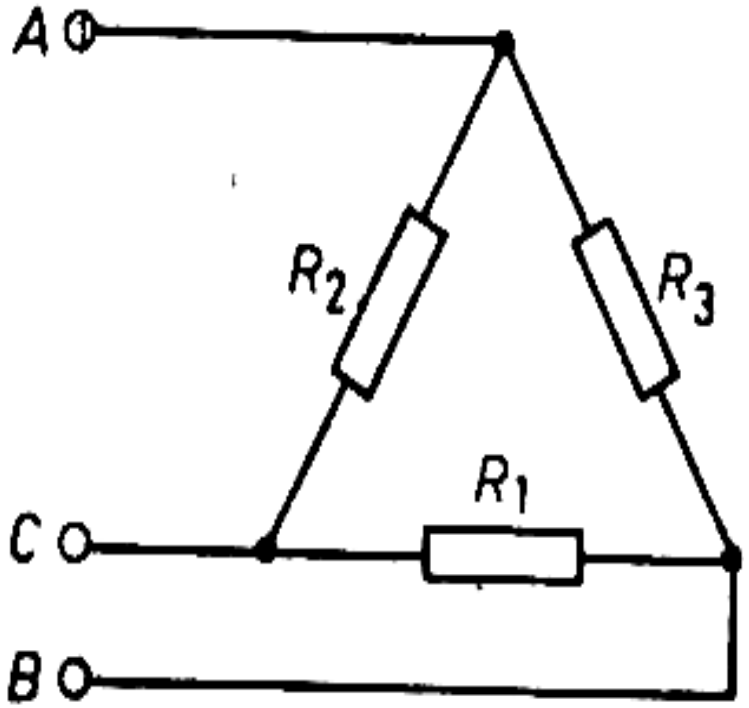
UNIT II

CIRCUIT ANALYSIS

Star-Delta Transformation



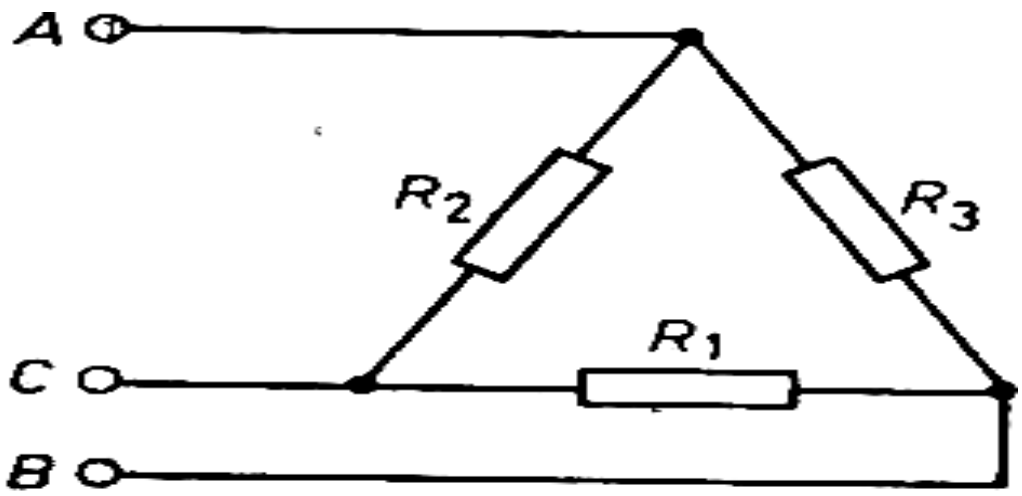
(a) Star (Y) section



(b) Delta or mesh (Δ) section

Equivalence

- **Equivalence can be found on the basis that the resistance between any pair of terminals in the two circuits have to be the same, when the third terminal is left open.**



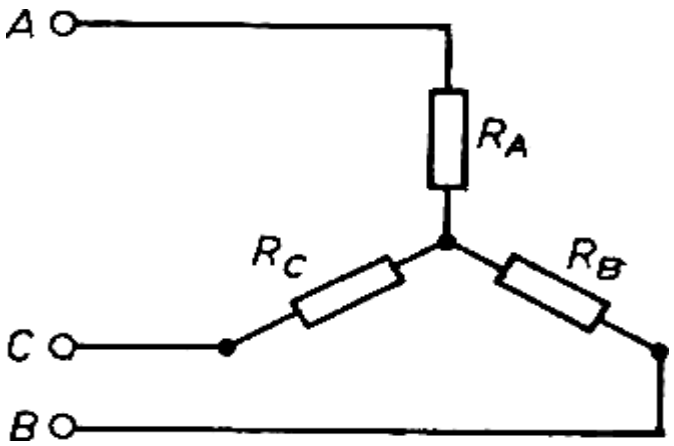
(b) Delta or mesh (Δ) section

- First take delta connection: between A and C, there are two parallel paths, one having a resistance of R_2 and other having a resistance of (R_1+R_3)

Hence resistance between terminal A and C is

$= R_2.(R_1+R_3)/[R_2+(R_1+R_3)]$

- Now take the star connection



The resistance between the same terminalA and C is (R_A+R_C)

Since terminal resistance have to be same so we must have

$$(R_A+R_C) = R_2.(R_1+R_3)/[R_2+(R_1+R_3)] \qquad (1)$$

Similarly for terminals A and B, B and C, we can have the following expression

$$(R_A+R_B) = R_3.(R_1+R_2)/[R_3+(R_1+R_2)] \qquad (2)$$
$$(R_B+R_C) = R_1.(R_2+R_3)/[R_1+(R_2+R_3)] \qquad (3)$$

DELTA to STAR

Now subtracting 2 from 1 and adding the result to 3, we will get the following values for R_1,R_2 and R_3.

$$R_A = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

How to remember?
Resistance of each arm of star is given by the product of the resistance of the two delta sides that meet at its ends divided by the sum of the three delta resistance

STAR to DELTA

Multiplying 1 and 2, 2 and 3 , 3 and 1 and adding them together and simplifying, we will have the following result.

$$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

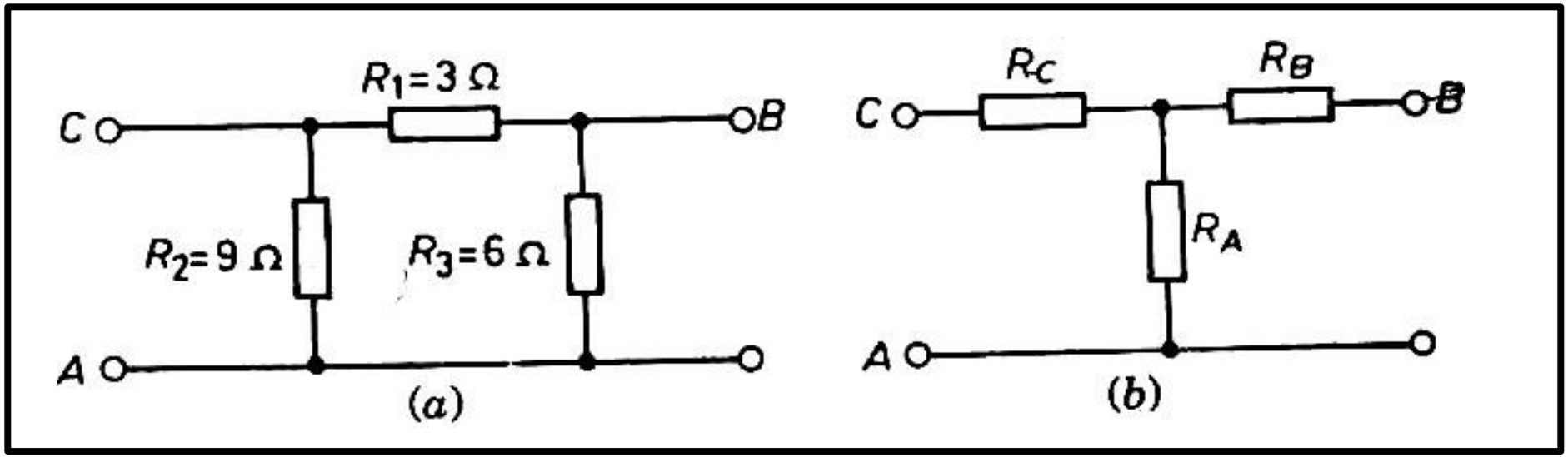
$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

How to remember: The equivalent delta resistance between any two point is given by the product of resistance taken two at a time divided by the opposite resistance in the star configuration.

Problem

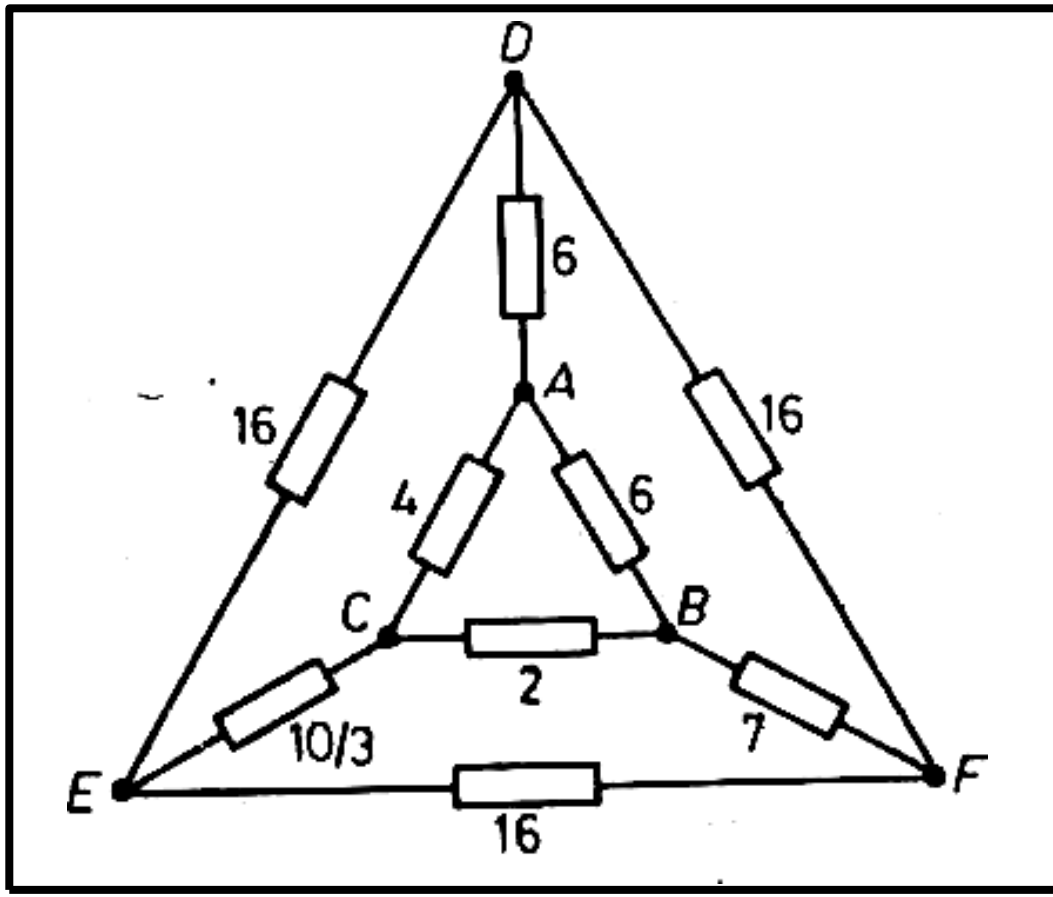
- **A delta-section of resistors is given in figure. Convert this into an equivalent star-section.**



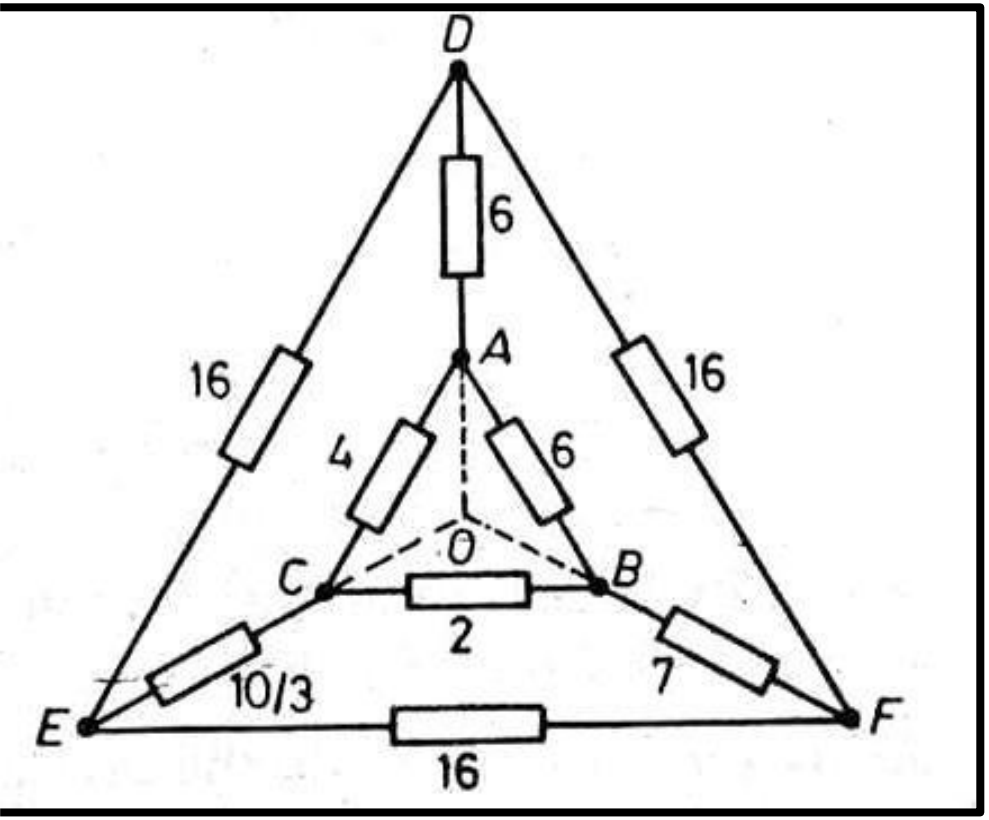
Ans . : $R_A = 3\ \Omega$; $R_B = 1.0\ \Omega$; $R_C = 1.5\ \Omega$.

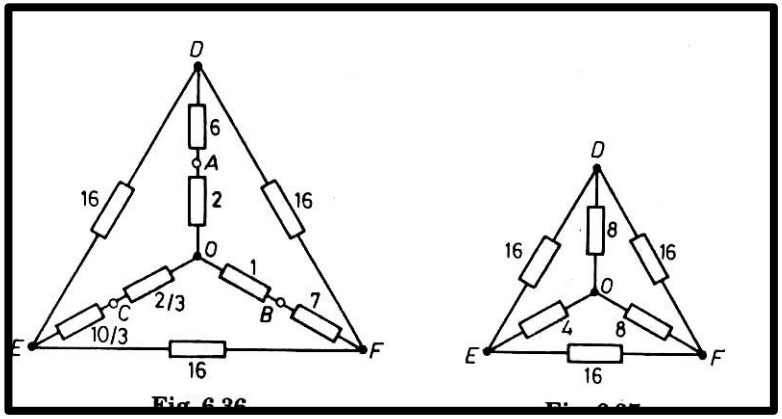
Problem

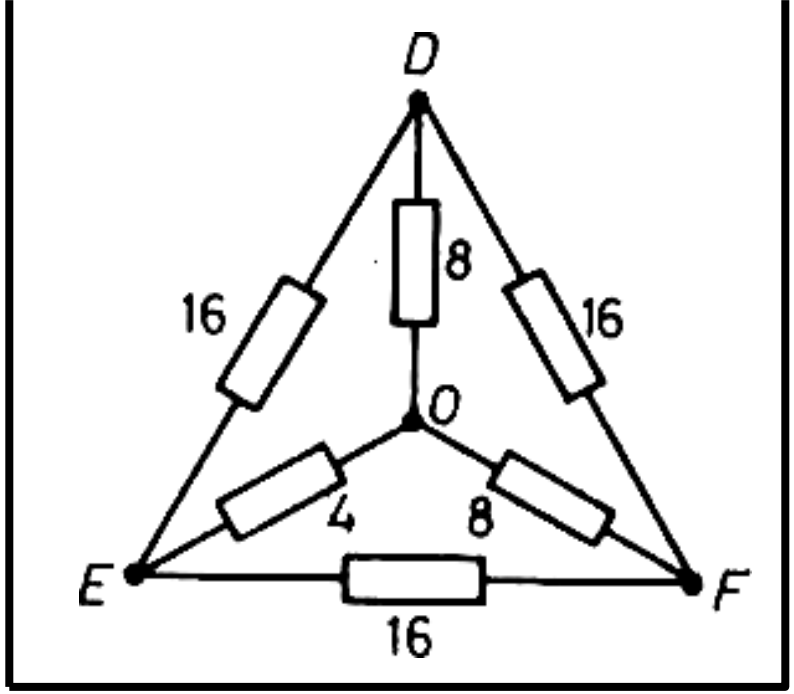
The figure shows a network. The number on each branch represents the value of resistance in ohms. Find the resistance between the points *E* and *F.*



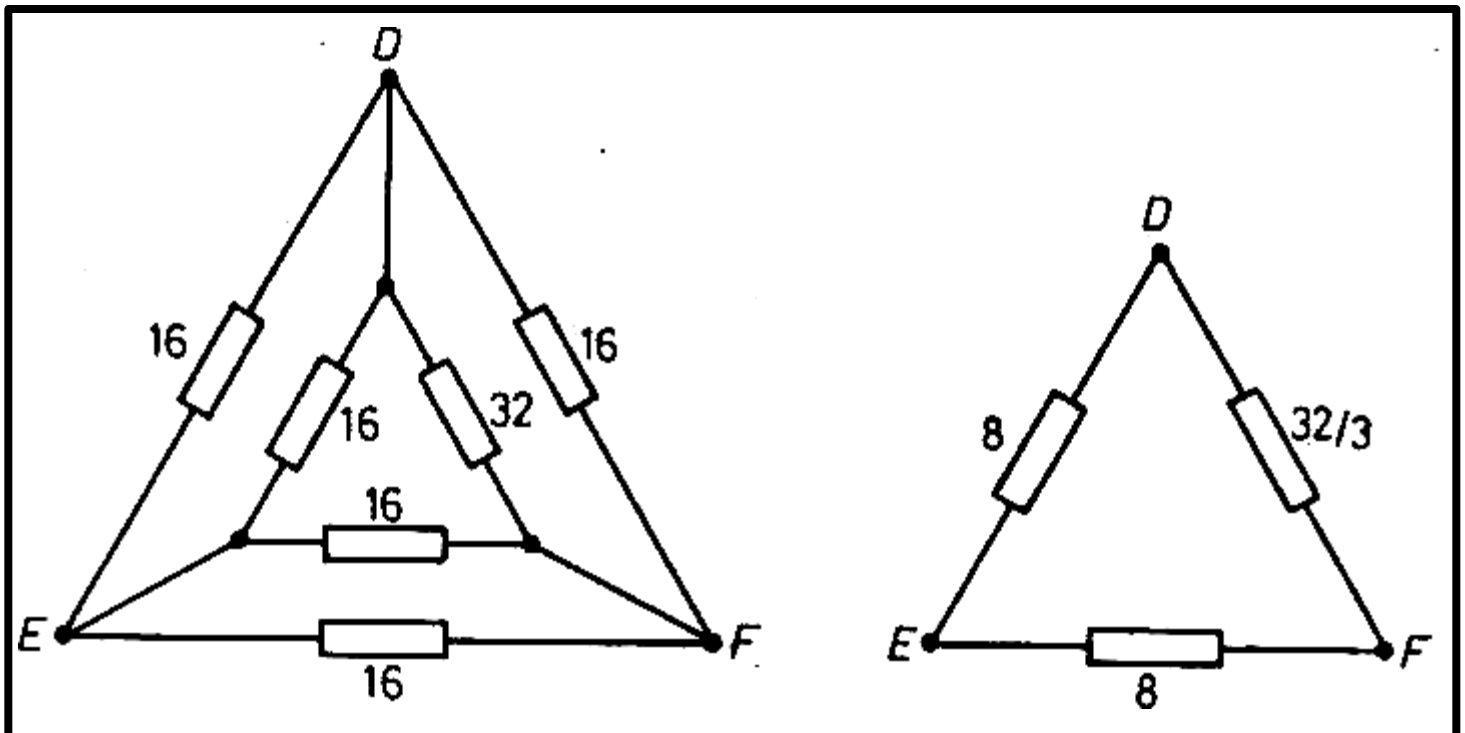
Solution


D
6
A
16
16
4
6
O
C
B
2
10/3
7
E
F
16


D
6
A
16
16
2
O
1
2/3
C
B
7
10/3
E
F
16
D
8
16
16
O
4
8
E
F
16

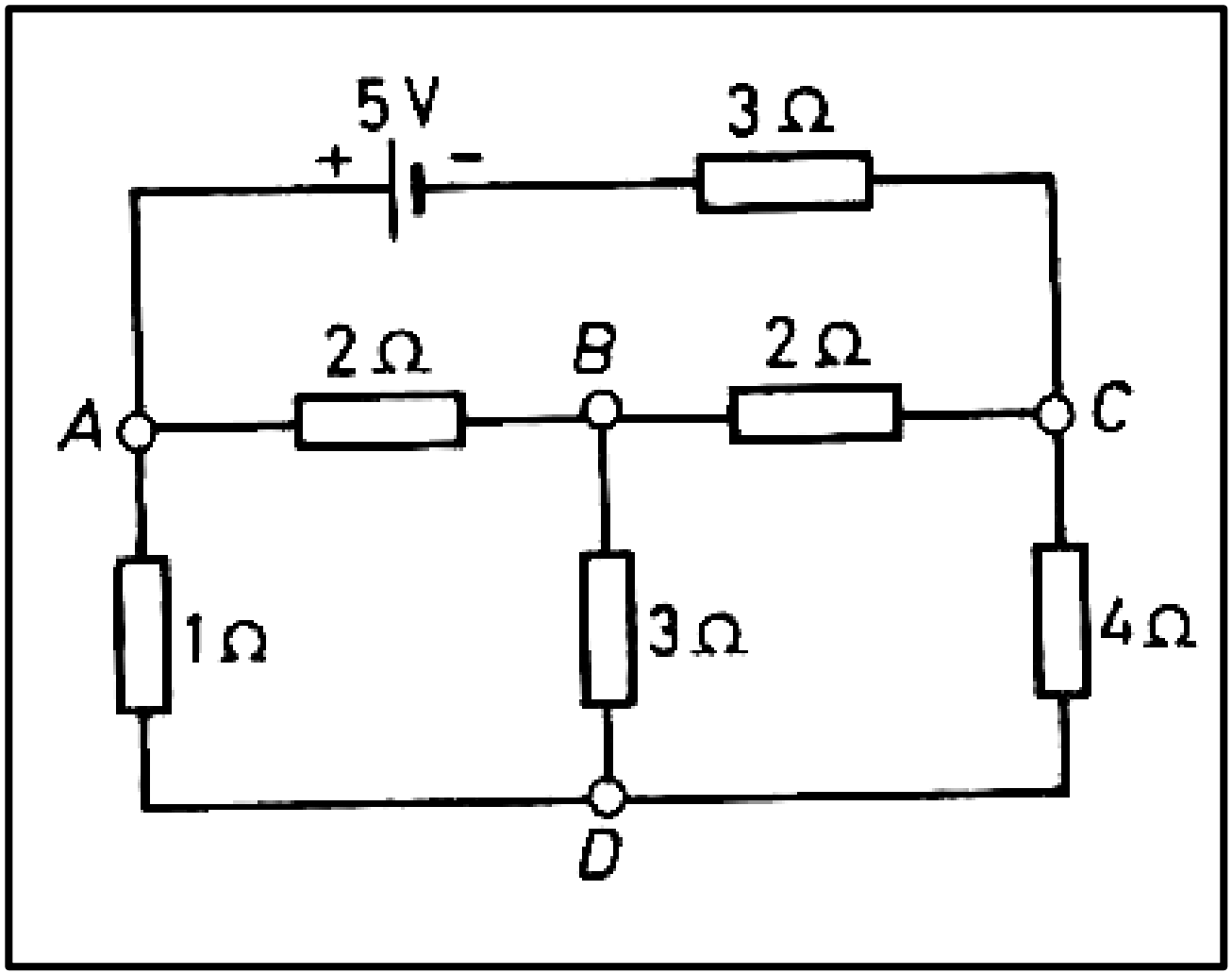


Ans. : 5.6 Ω

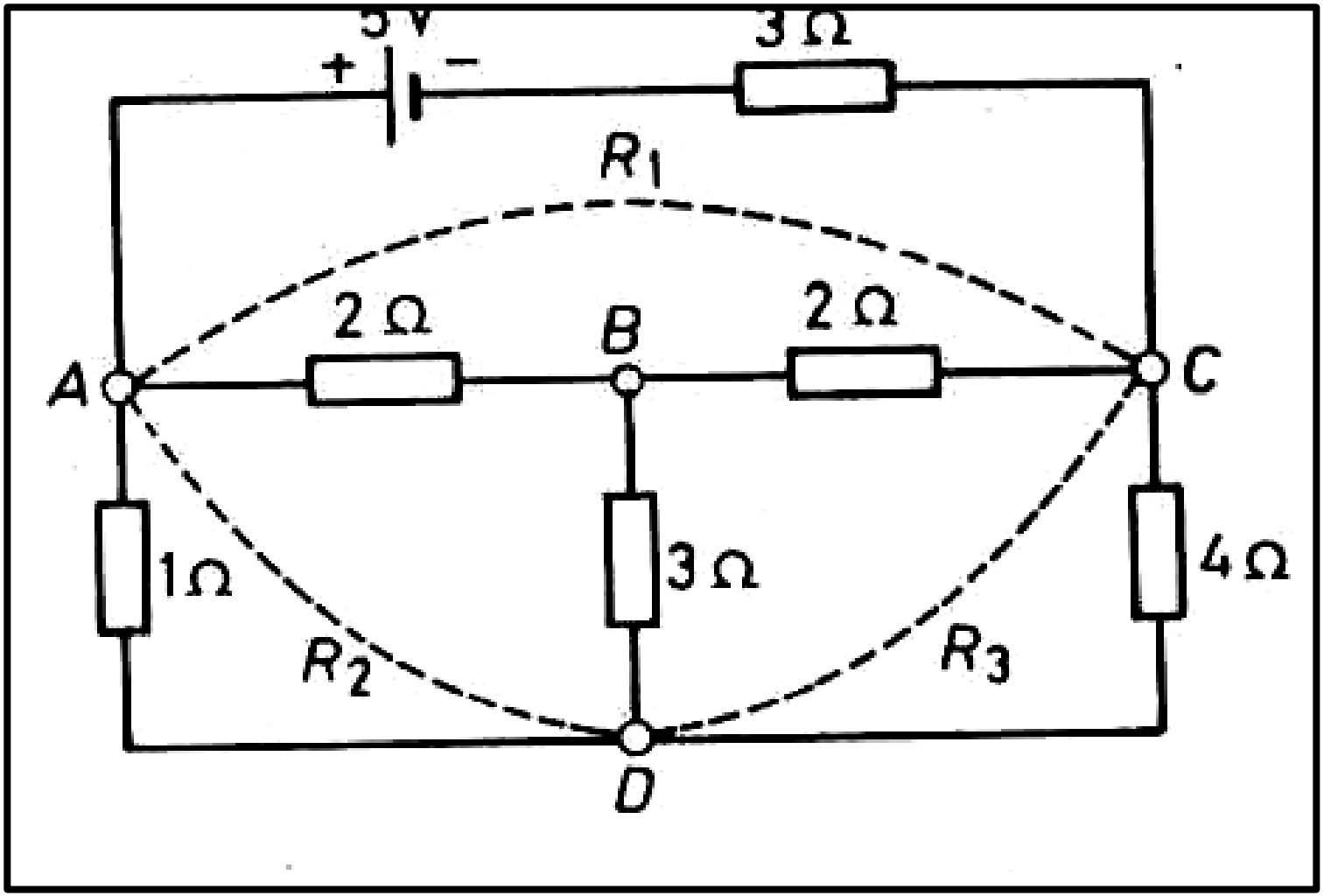


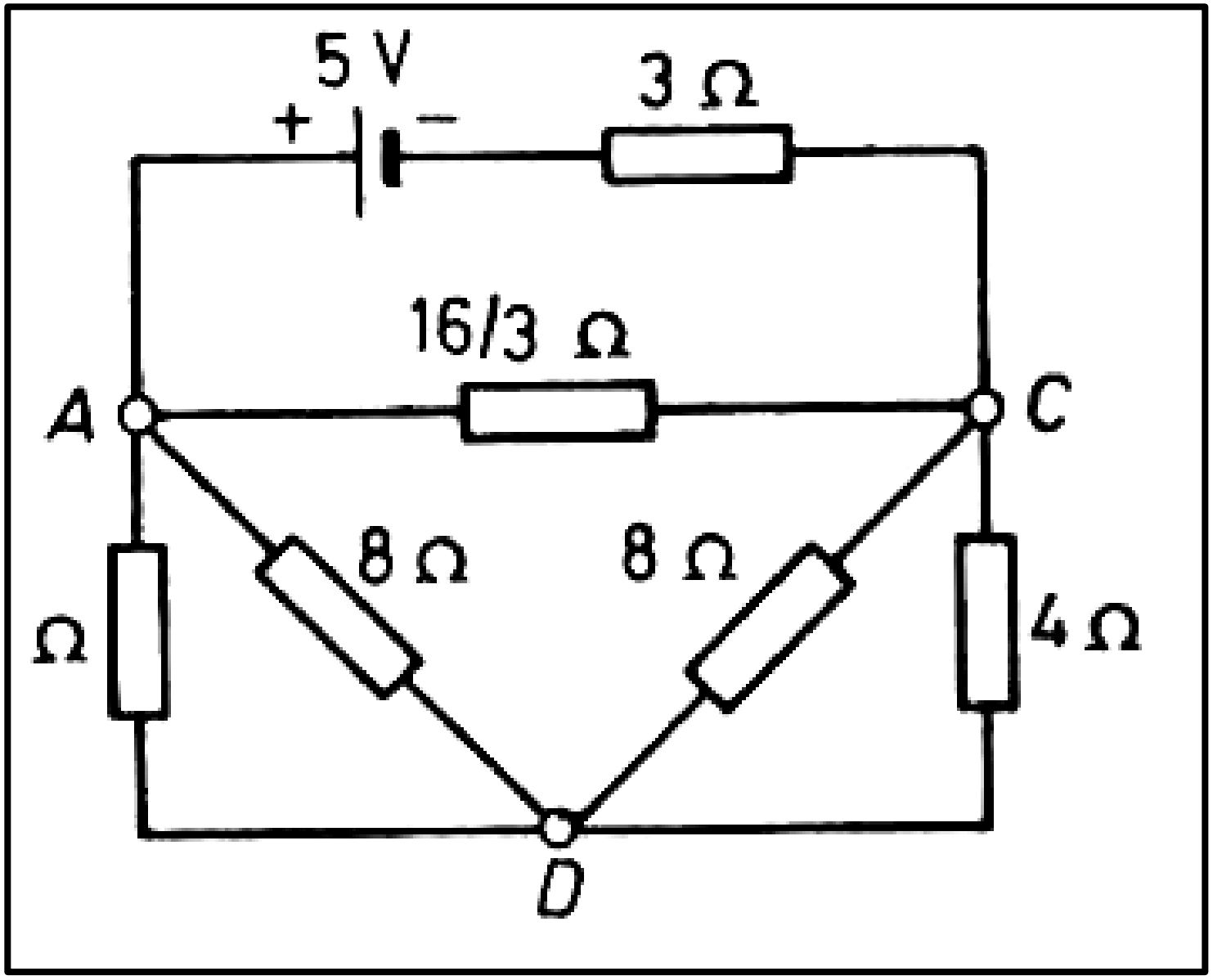
Problem

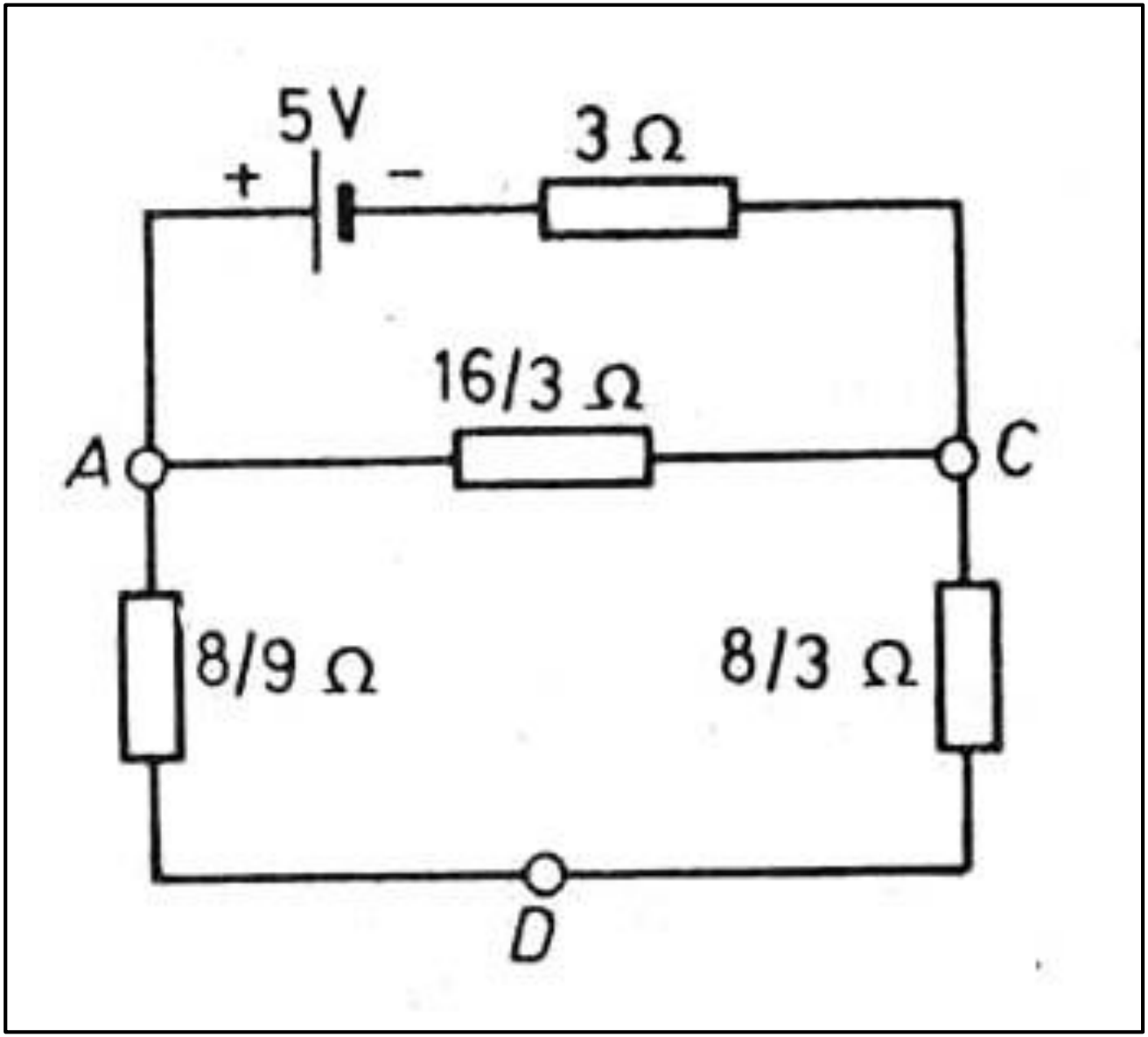
- Find the current drawn from the 5 volt battery in the network shown in figure.

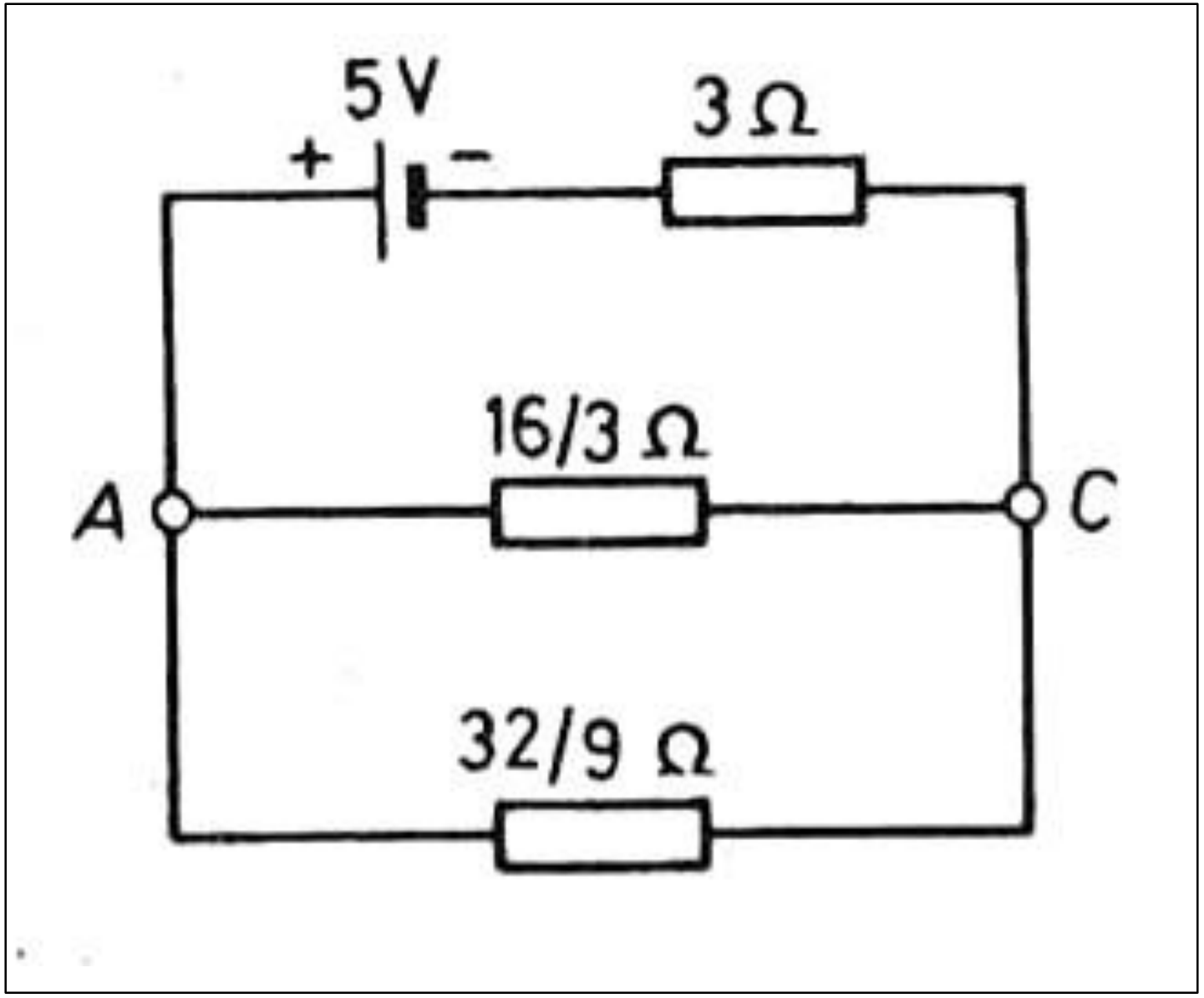


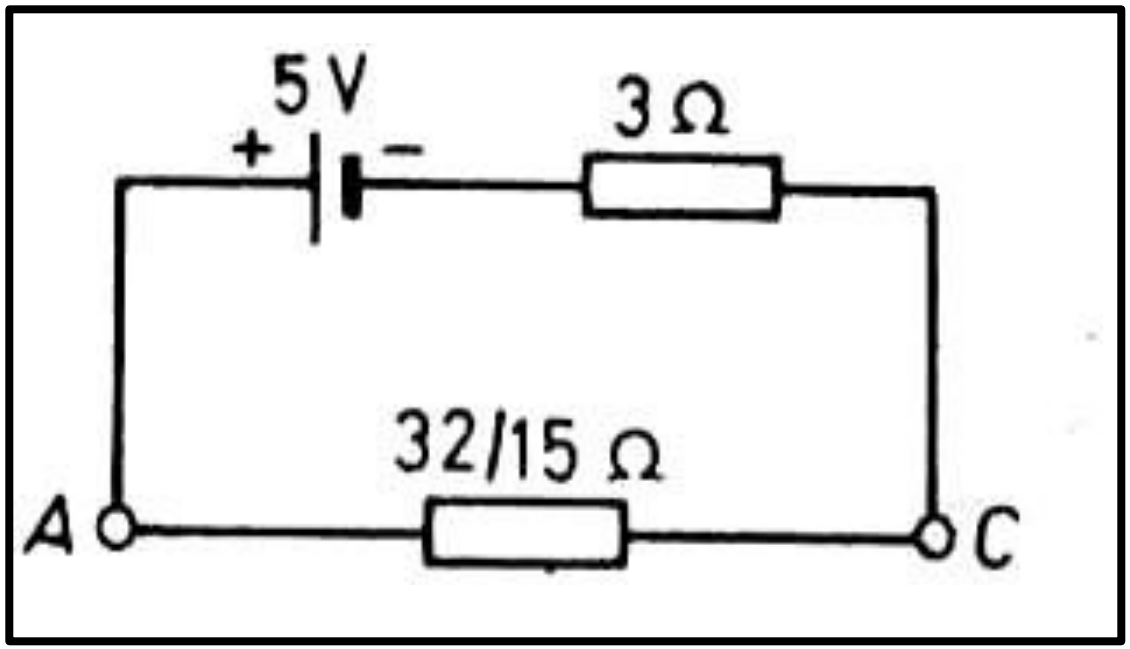
Solution :



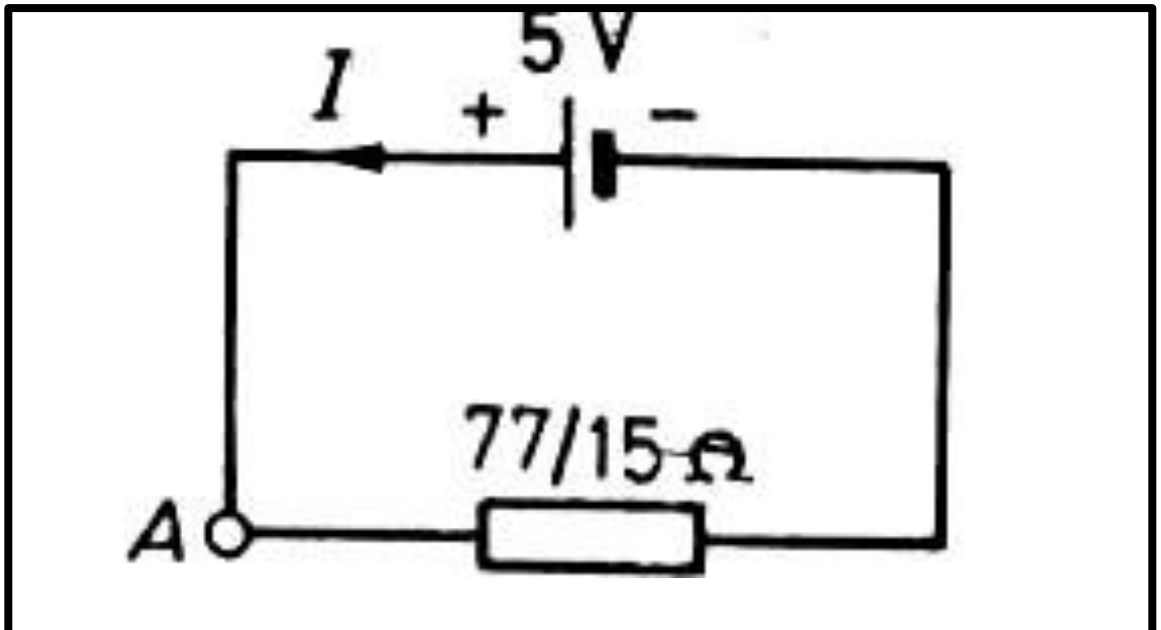

5 V
+
−
3 Ω
16/3 Ω
A
C
8 Ω
8 Ω
Ω
4 Ω
D


5 V
+
−
3 Ω
16/3 Ω
A
C
8/9 Ω
8/3 Ω
D


5 V
+
−
3 Ω
16/3 Ω
A
C
32/9 Ω



Ans. : 0.974 A



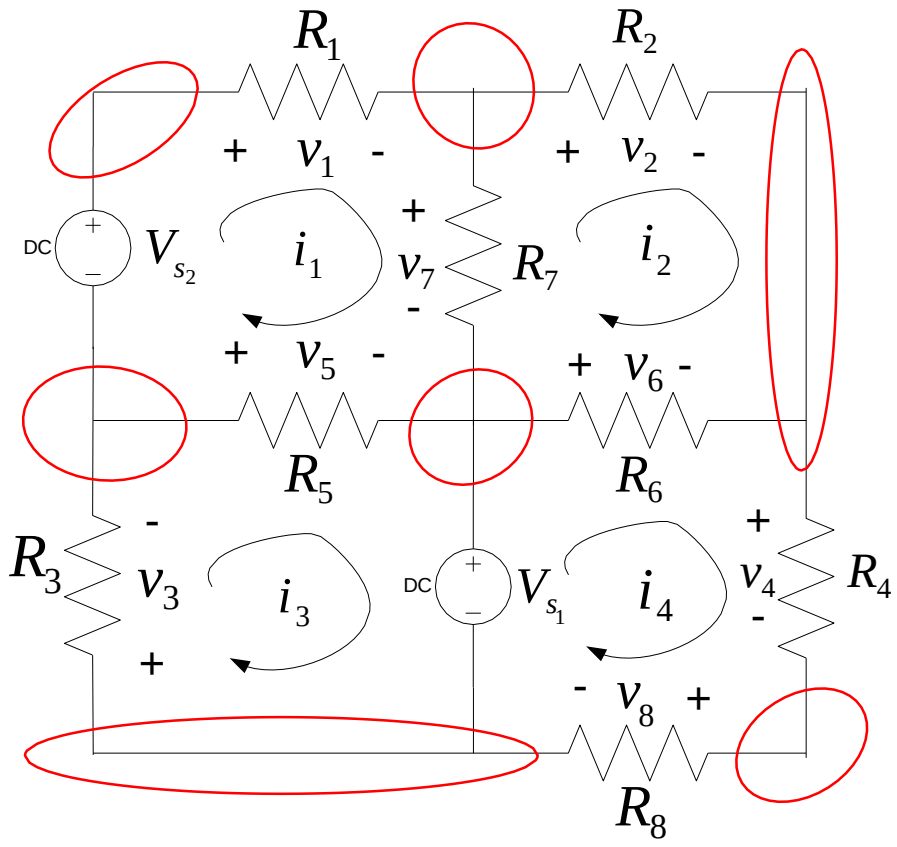
Mesh Analysis

- Mesh analysis applies KVL to find unknown currents.
- It is only applicable to **planar** circuits (a circuit that can be drawn on a plane with no branches crossing each other).
- A mesh is a loop that does not contain any other loops.
- The current through a mesh is known as the mesh current.
- Assume for simplicity that the circuit contains only voltage sources.

Mesh Analysis Steps

1. Assign mesh currents i_1, i_2, i_3, …i_l, to the l meshes,
2. Apply KVL to each of the l meshes and use Ohm's law to express the voltages in terms of the mesh currents,
3. Solve the l resulting simultaneous equations to find the mesh currents.

Example



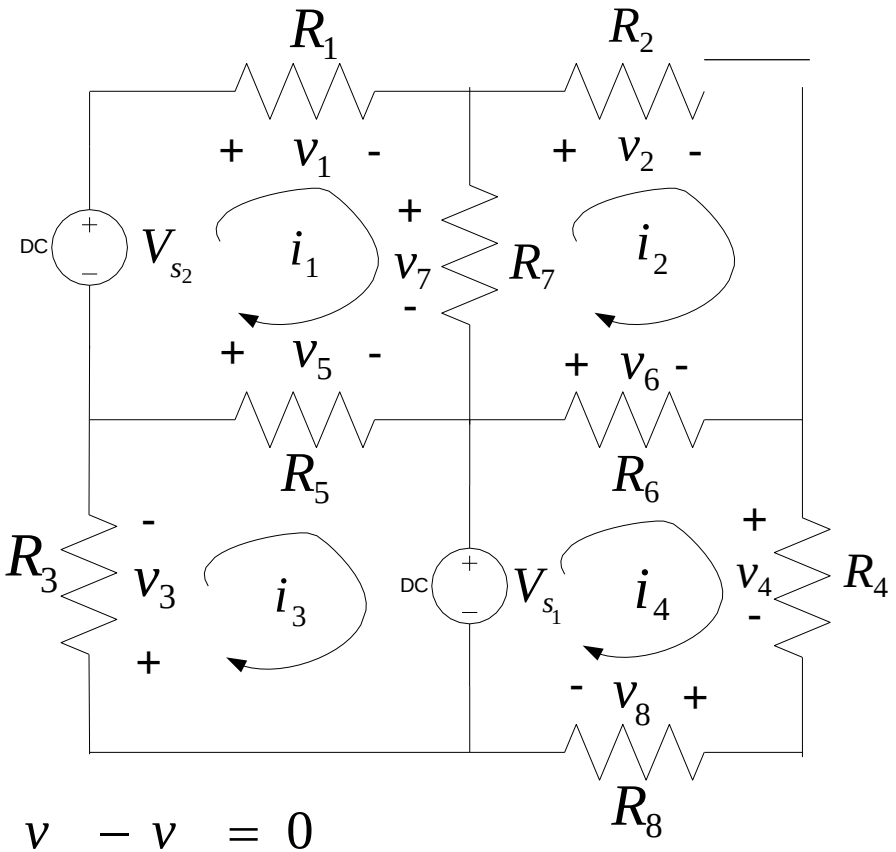
Number of nodes, n = **7**

Number of loops, l = **4**

Number of branches, b = **10**

$$l = b - n + 1$$

Example



Apply KVL to each mesh

Mesh 1: $-V_{s_2} + v_1 + v_7 - v_5 = 0$

Mesh 2: $v_2 - v_6 - v_7 = 0$

Mesh 3: $v_5 + v_{s_1} + v_3 = 0$

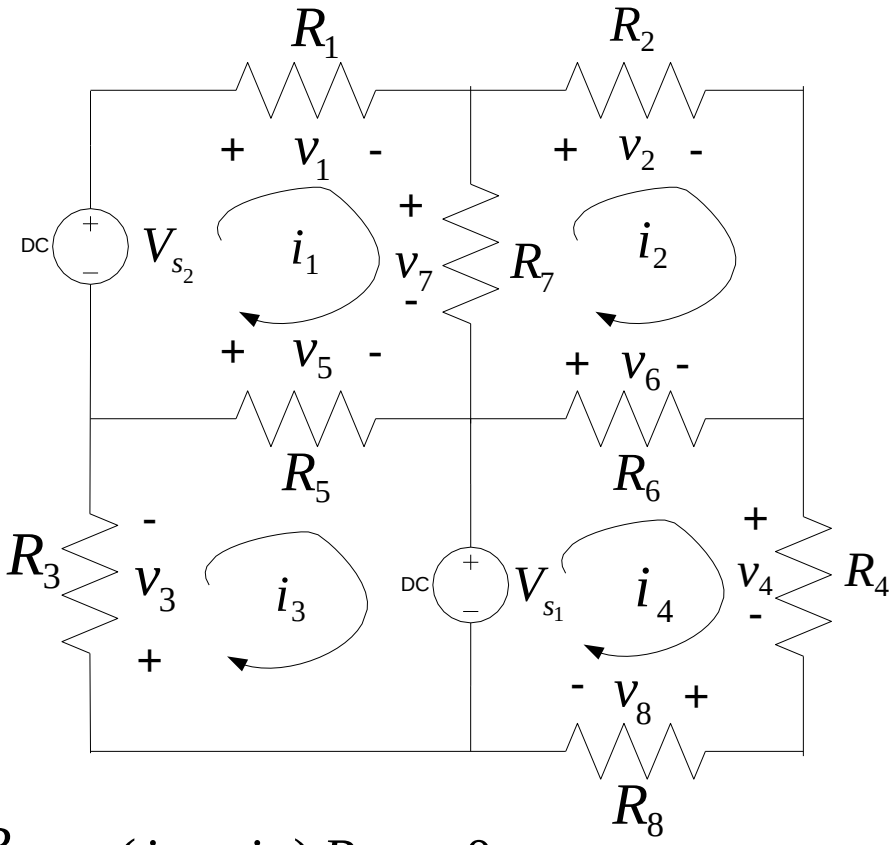
Mesh 4: $v_4 + v_8 - V_{s_1} + v_6 = 0$

Mesh 1: $-V_{s_2} + v_1 + v_7 - v_5 = 0$

Mesh 2: $v_2 - v_6 - v_7 = 0$

Mesh 3: $v_5 + v_{s_1} + v_3 = 0$

Mesh 4: $v_4 + v_8 - V_{s_1} + v_6 = 0$



Express the voltage in terms of the mesh currents:

Mesh 1: $-V_{s_2} + i_1 R_1 + (i_1 - i_2) R_7 + (i_1 - i_3) R_5 = 0$

Mesh 2: $i_2 R_2 + (i_2 - i_4) R_6 + (i_2 - i_1) R_7 = 0$

Mesh 3: $(i_3 - i_1) R_5 + V_{s_1} + i_3 R_3 = 0$

Mesh 4: $i_4 R_4 + i_4 R_8 - V_{s_1} + (i_4 - i_2) R_6 = 0$

Mesh 1: $-V_{s_2} + iR_1 + (i_1 - i_2)R_7 + (i_1 - i_3)R_5 = 0$

Mesh 2: $i_2 R_2 + (i_2 - i_4)R_6 + (i_2 - i_1)R_7 = 0$

Mesh 3: $(i_3 - i_1)R_5 + V_{s_1} + i_3 R_3 = 0$

Mesh 4: $i_4 R_4 + i_4 R_8 - V_{s_1} + (i_4 - i_2)R_6 = 0$

Mesh 1: $(R_1 + R_5 + R_7)i_1 - R_7 i_2 - R_5 i_3 = V_{s_2}$

Mesh 2: $-R_7 i_1 + (R_2 + R_6 + R_7)i_2 - R_6 i_4 = 0$

Mesh 3: $-R_5 i_1 + (R_3 + R_5)i_3 = -V_{s_1}$

Mesh 4: $-R_6 i_2 + (R_4 + R_6 + R_8)i_4 = V_{s_1}$

Mesh 1: $$(R_1 + R_5 + R_7)i_1 - R_7 i_2 - R_5 i_3 = V_{s_2}$$

Mesh 2: $$-R_7 i_1 + (R_2 + R_6 + R_7)i_2 - R_6 i_4 = 0$$

Mesh 3: $$-R_5 i_1 + (R_3 + R_5)i_3 = -V_{s_1}$$

Mesh 4: $$-R_6 i_2 + (R_4 + R_6 + R_8)i_4 = V_{s_1}$$

$$\begin{pmatrix} R_1 + R_5 + R_7 & -R_7 & -R_5 & 0 \\ -R_7 & R_2 + R_6 + R_7 & 0 & -R_6 \\ -R_5 & 0 & R_3 + R_5 & 0 \\ 0 & -R_6 & 0 & R_4 + R_6 + R_8 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} V_{s_2} \\ 0 \\ -V_{s_1} \\ V_{s_1} \end{pmatrix}$$

$$\begin{pmatrix} R_1 + R_5 + R_7 & -R_7 & -R_5 & 0 \\ -R_7 & R_2 + R_6 + R_7 & 0 & -R_6 \\ -R_5 & 0 & R_3 + R_5 & 0 \\ 0 & -R_6 & 0 & R_4 + R_6 + R_8 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} V_{s_2} \\ 0 \\ -V_{s_1} \\ V_{s_1} \end{pmatrix}$$

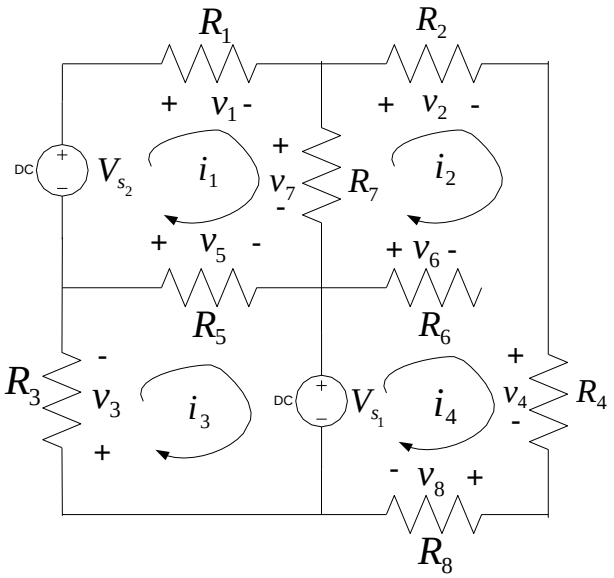
$\mathbf{R\,i = v}$

R is an *l* x *l* *symmetric* resistance matrix

i is a 1 x *l* vector of mesh currents

v is a vector of voltages representing —known|| voltages

Writing the Mesh Equations by Inspection



$$\begin{pmatrix} R_1+R_5+R_7 & -R_7 & -R_5 & 0 \\ -R_7 & R_2+R_6+R_7 & 0 & -R_6 \\ -R_5 & 0 & R_3+R_5 & 0 \\ 0 & -R_6 & 0 & R_4+R_6+R_8 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} V_{s_2} \\ 0 \\ -V_{s_1} \\ V_{s_1} \end{pmatrix}$$

- The matrix **R** is symmetric, $r_{kj} = r_{jk}$ and all of the off-diagonal terms are negative or zero.

The r_{kk} terms are the sum of all resistances in mesh k.

The r_{kj} terms are the negative sum of the resistances common to BOTH mesh k and mesh j.

The v_k (the k^{th} component of the vector **v**) = the algebraic sum of the independent voltages in mesh k, with voltage rises taken as positive.

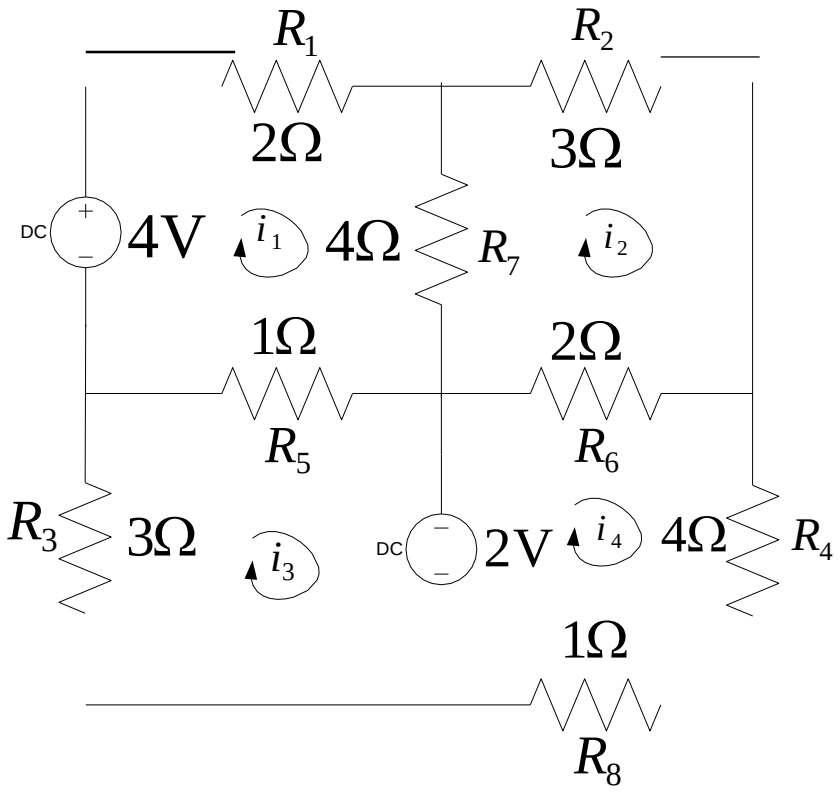
MATLAB Solution of Mesh Equations

$$\begin{pmatrix} R_1 + R_5 + R_7 & -R_7 & -R_5 & 0 \\ -R_7 & R_2 + R_6 + R_7 & 0 & -R_6 \\ -R_5 & 0 & R_3 + R_5 & 0 \\ 0 & -R_6 & 0 & R_4 + R_6 + R_8 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} V_{s_2} \\ 0 \\ -V_{s_1} \\ V_{s_1} \end{pmatrix}$$

$$\mathbf{R\,i} = \mathbf{v}$$

$$\mathbf{i} = \mathbf{R}^{-1}\mathbf{v}$$

Test with numbers



$$\begin{pmatrix} 2+4+1 & -4 & -1 & 0 \\ -4 & 3+2+4 & 0 & -2 \\ -1 & 0 & 3+1 & 0 \\ 0 & -2 & 0 & 2+4+1 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -2 \\ 2 \end{pmatrix}$$

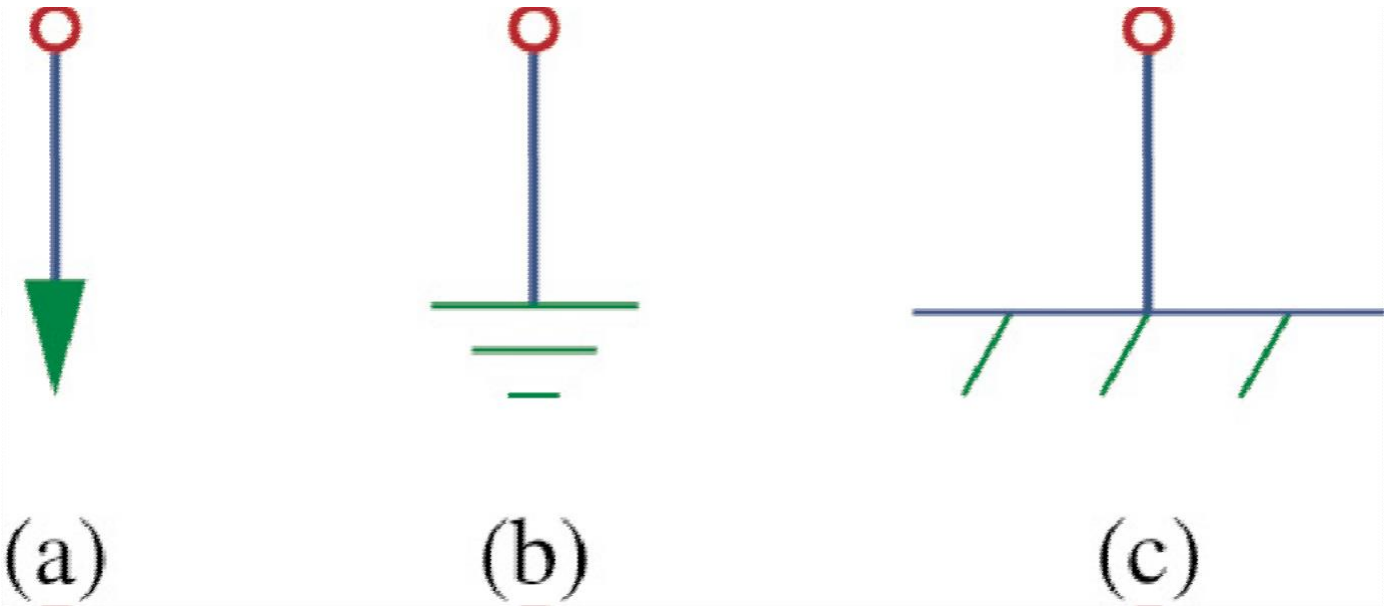
Test with numbers

$$\begin{pmatrix} 2+4+1 & -4 & -1 & 0 \\ -4 & 3+2+4 & 0 & -2 \\ -1 & 0 & 3+1 & 0 \\ 0 & -2 & 0 & 2+4+1 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -2 \\ 2 \end{pmatrix}$$

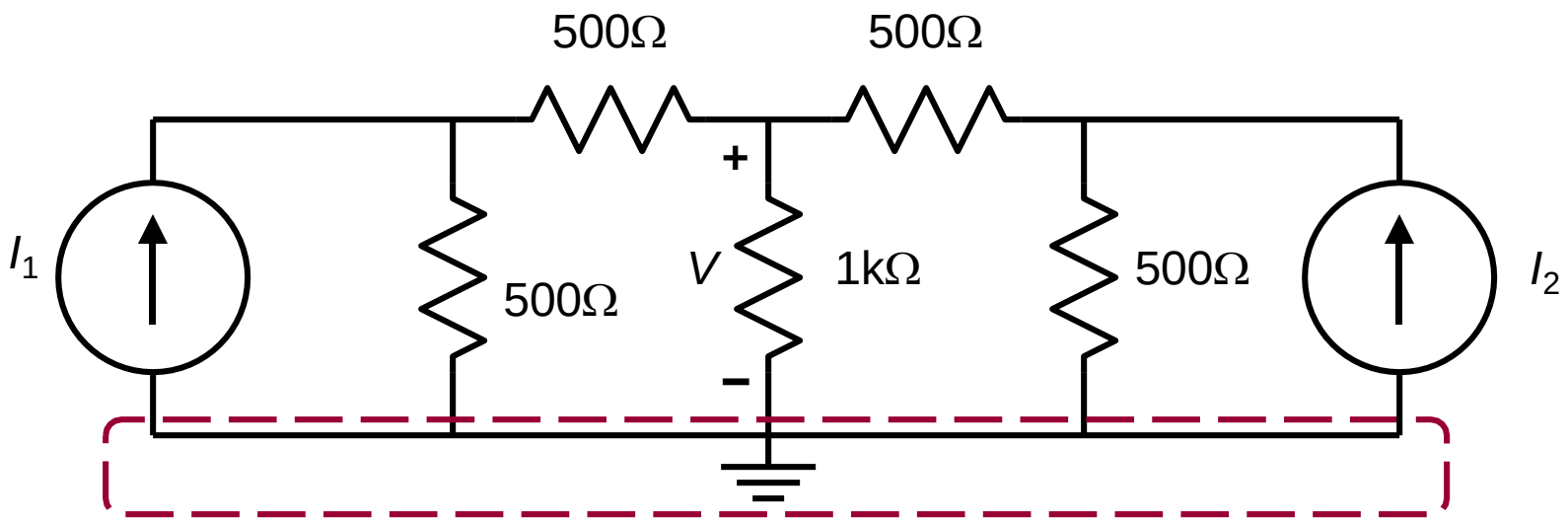
$$\begin{pmatrix} 7 & -4 & -1 & 0 \\ -4 & 9 & 0 & -2 \\ -1 & 0 & 4 & 0 \\ 0 & -2 & 0 & 7 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -2 \\ 2 \end{pmatrix}$$

$$\mathbf{R\,i = v}$$

Common symbols for indicating a reference node,
(a) common ground, (b) ground, (c) chassis.

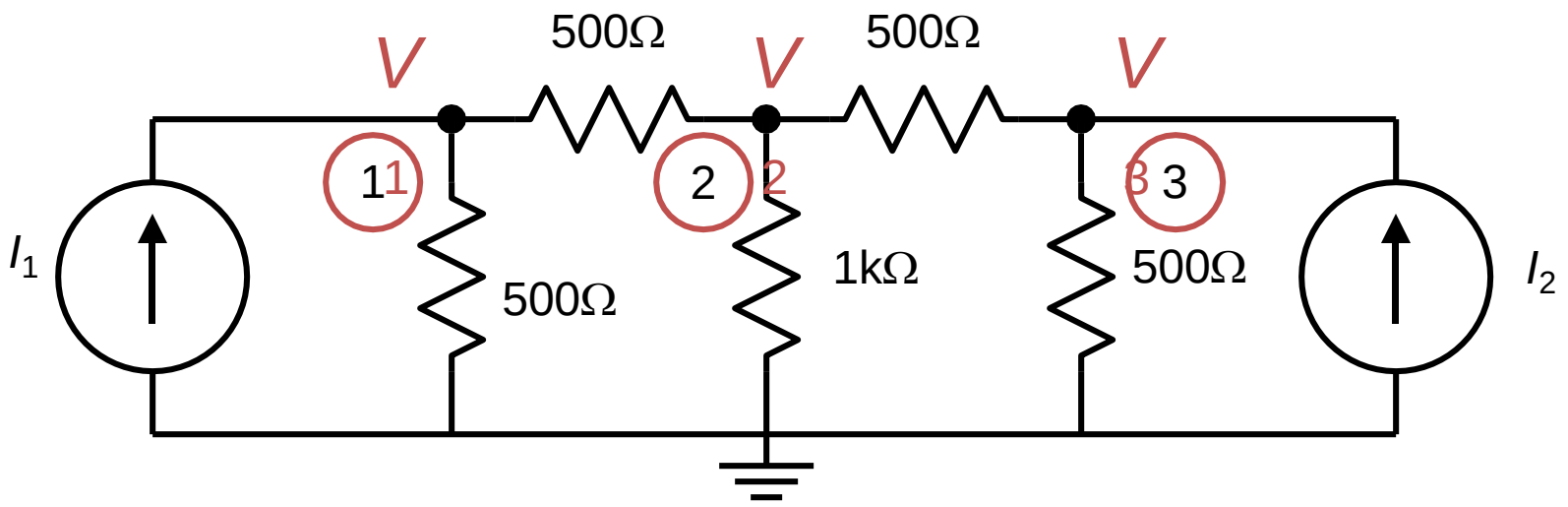


1. Reference Node



The reference node is called the *ground* node where $V = 0$

2. Node Voltages



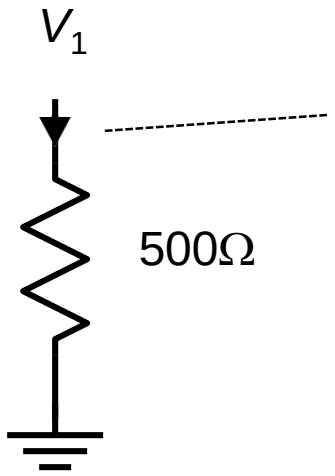
V_1, V_2, and V_3 are unknowns for which we solve using KCL

Steps of Nodal Analysis

1. Choose a reference (ground) node.
2. Assign node voltages to the other nodes.
3. Apply KCL to each node other than the reference node; express currents in terms of node voltages.
4. Solve the resulting system of linear equations for the nodal voltages.

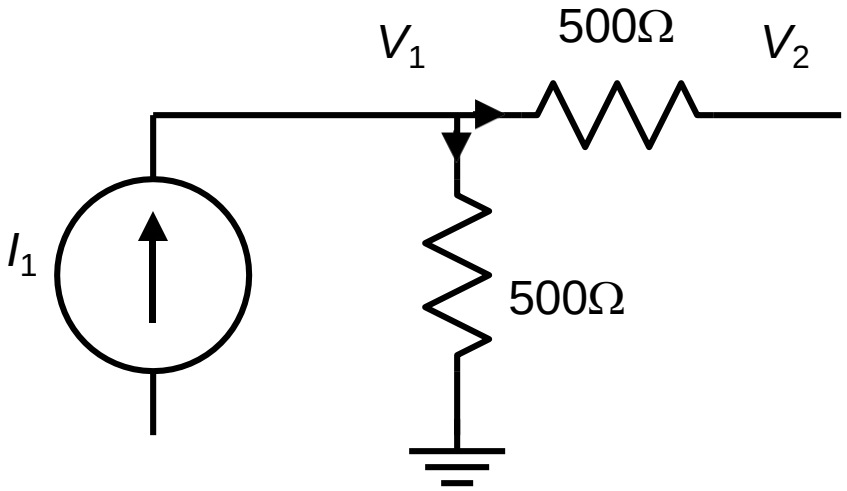
Currents and Node Voltages

$$\frac{V_1 - V_2}{500\ \Omega}$$



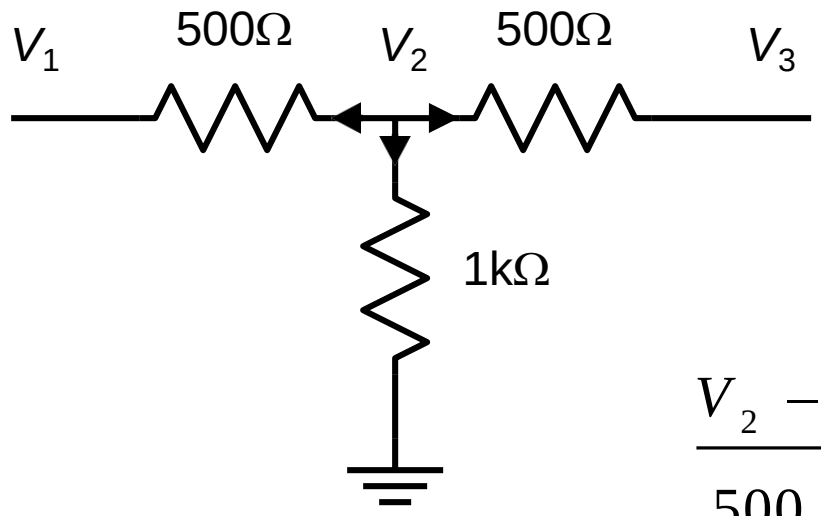
$$\frac{V_1}{500\ \Omega}$$

3. KCL at Node 1



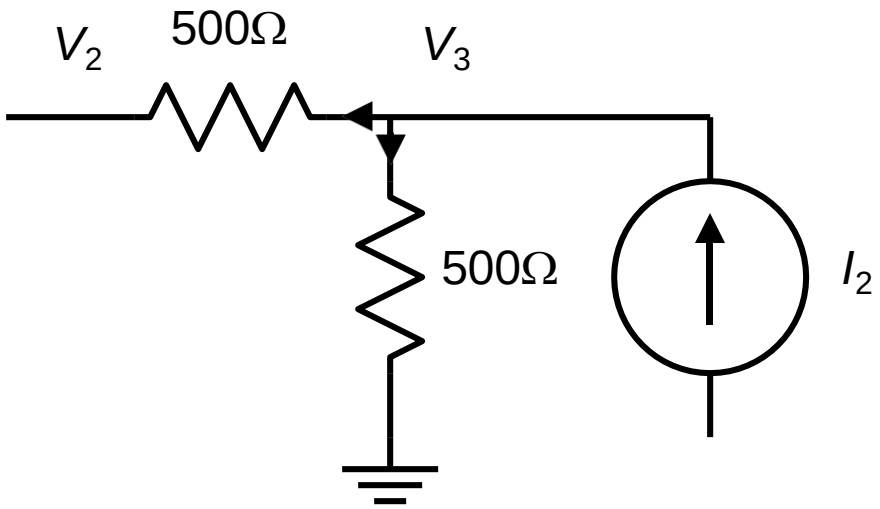
$$I_1 = \frac{V_1 - V_2}{500\ \Omega} + \frac{V_1}{500\ \Omega}$$

3. KCL at Node 2



$$\frac{V_2 - V_1}{500\ \Omega} + \frac{V_2}{1\text{k}\Omega} + \frac{V_2 - V_3}{500\ \Omega} = 0$$

3. KCL at Node 3

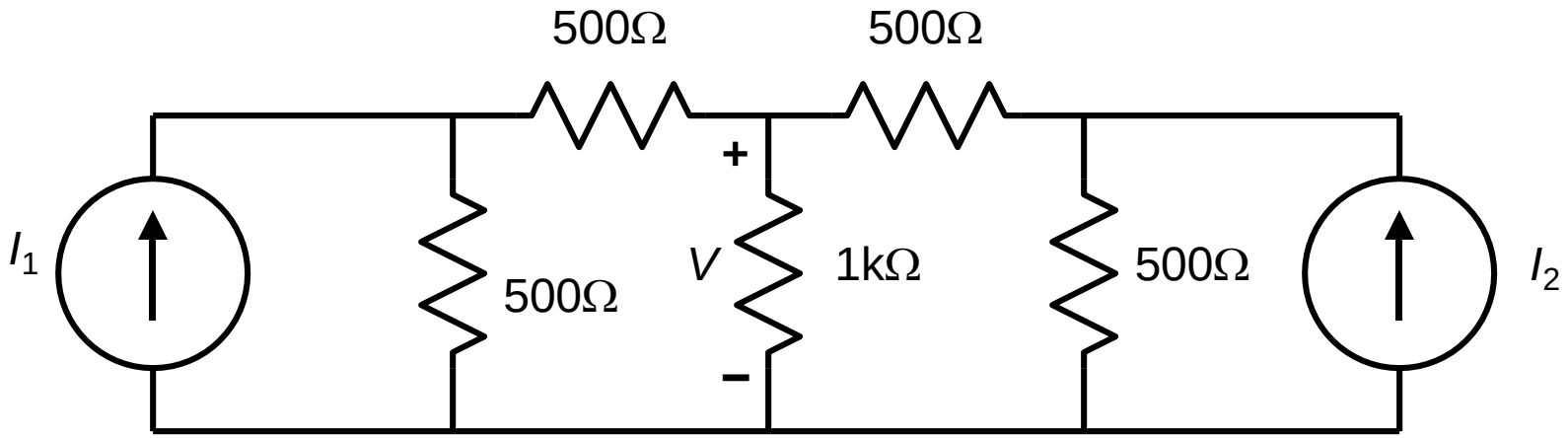


$$\frac{V_3 - V_2}{500\ \Omega} + \frac{V_3}{500\ \Omega} = I_2$$

Steps of Nodal Analysis

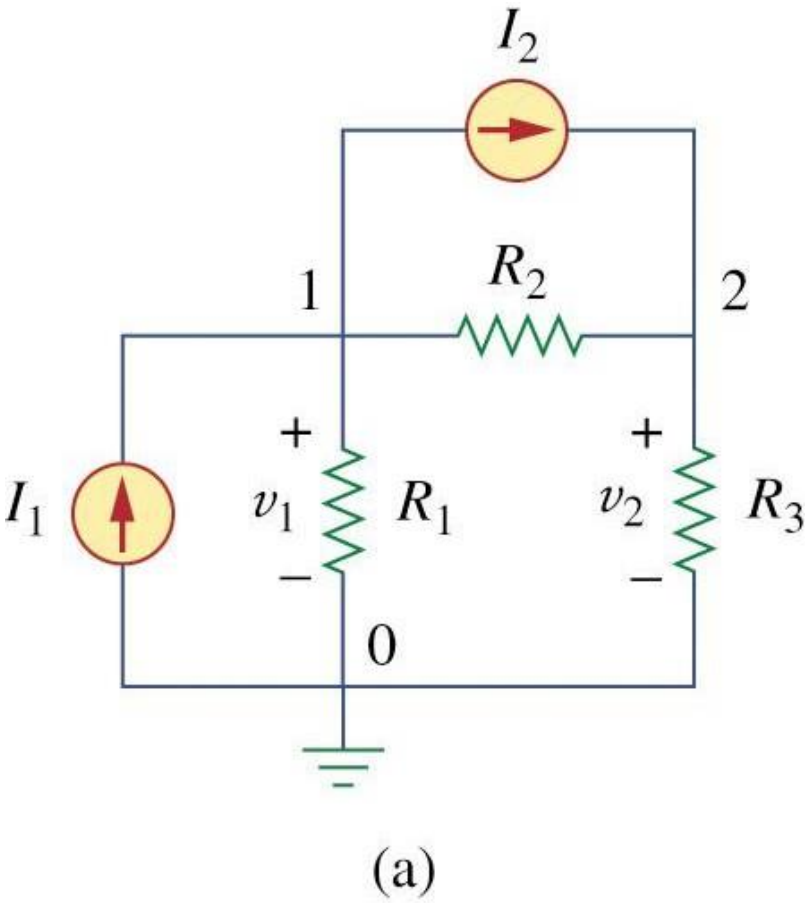
1. Choose a reference (ground) node.
2. Assign node voltages to the other nodes.
3. Apply KCL to each node other than the reference node; express currents in terms of node voltages.
4. Solve the resulting system of linear equations for the nodal voltages.

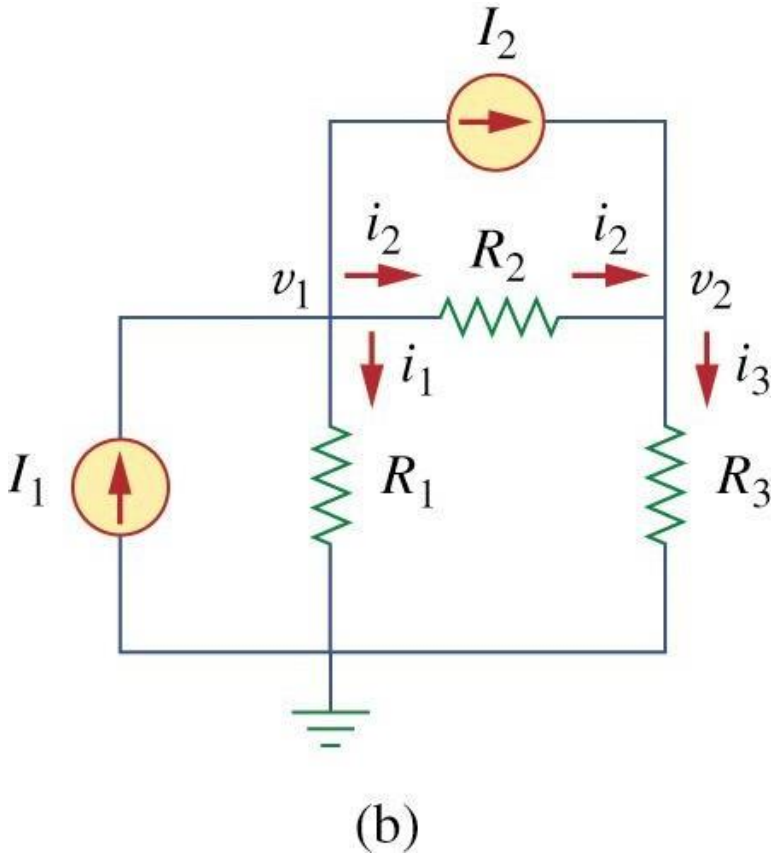
4. Summing Circuit Solution



Solution: $V = 167I_1 + 167I_2$

Typical circuit for nodal analysis





$$I_1 = I_2 + i_1 + i_2$$

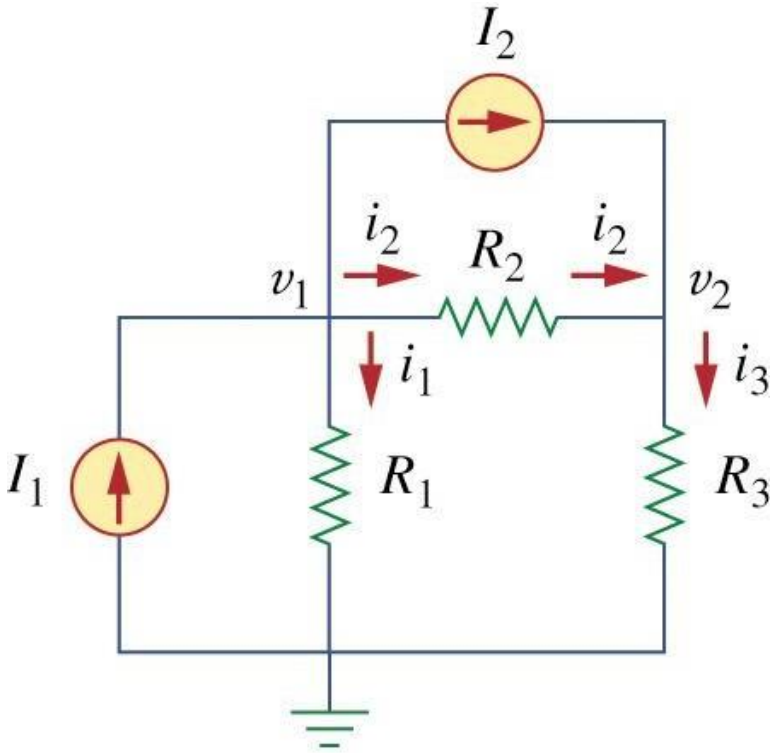
$$I_2 + i_2 = i_3$$

$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

$$i_1 = \frac{v_1 - 0}{R_1} \quad \text{or} \quad i_1 = G_1 v_1$$

$$i_2 = \frac{v_1 - v_2}{R_2} \quad \text{or} \quad i_2 = G_2 (v_1 - v_2)$$

$$i_3 = \frac{v_2 - 0}{R_3} \quad \text{or} \quad i_3 = G_3 v_2$$



(b)

$$\Rightarrow I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

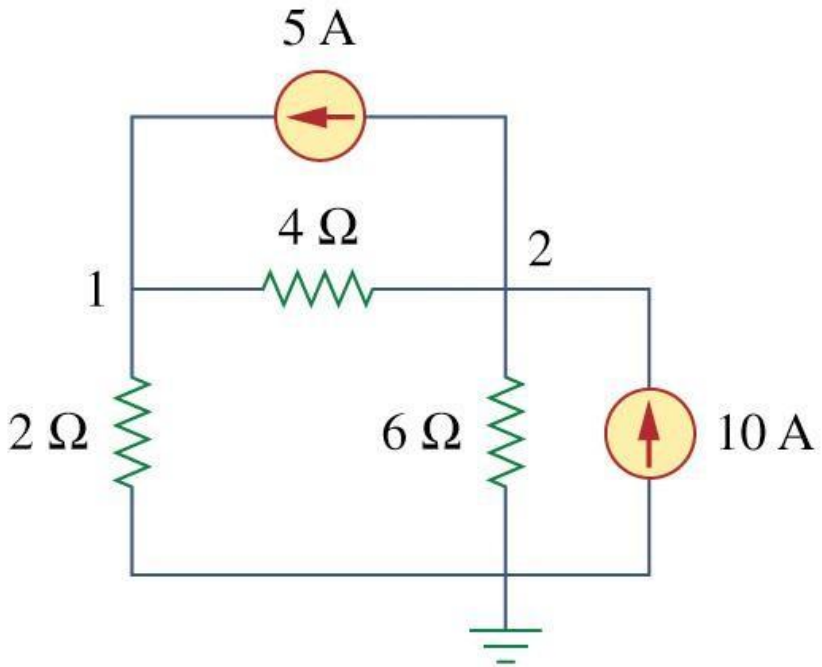
$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

$$\Rightarrow I_1 - I_2 = G_1 v_1 + G_2 (v_1 - v_2)$$

$$I_2 = -G_2 (v_1 - v_2) + G_3 v_2$$

$$\Rightarrow \begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

- Calculus the node voltage in the circuit shown in Fig. 3.3(a)

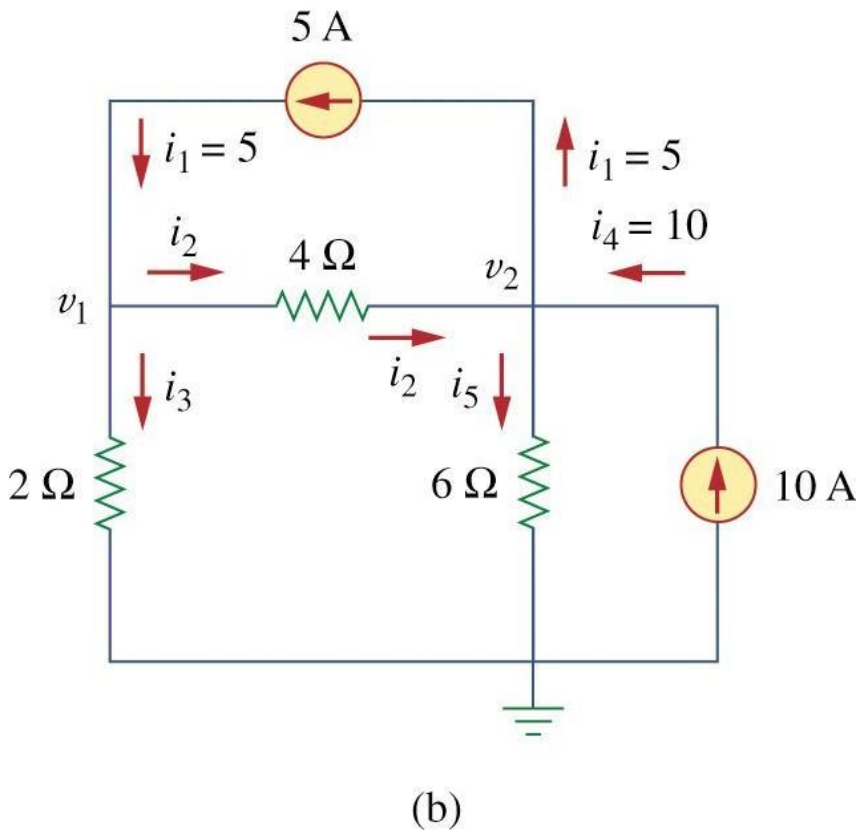


(a)

- At node 1

$$i_1 = i_2 + i_3$$

$$\Rightarrow 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

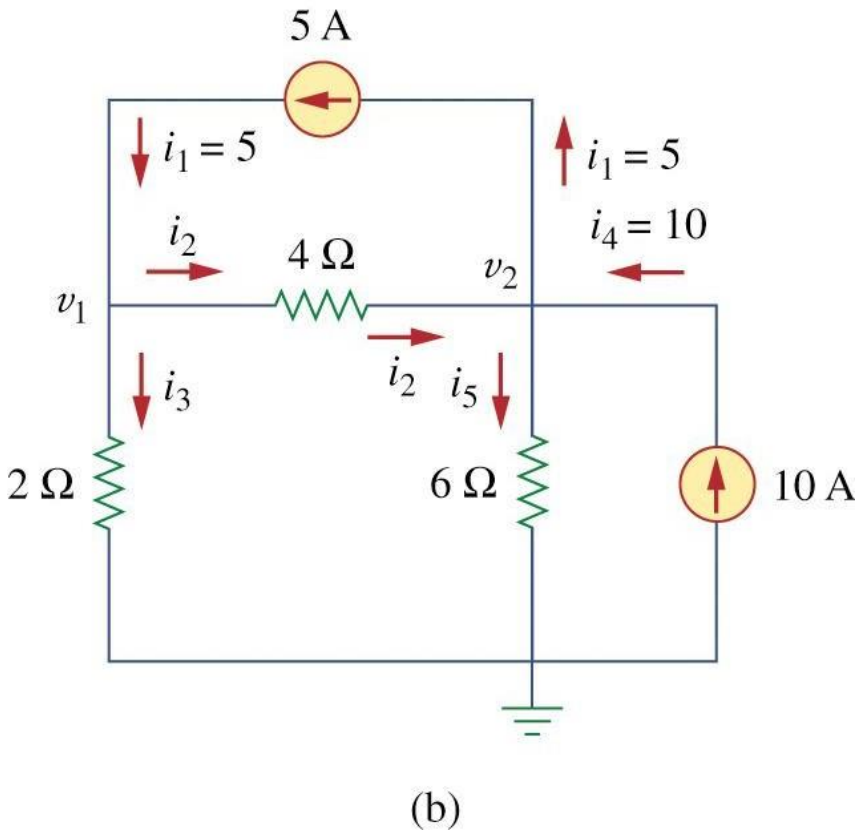


(b)

- At node 2

$$i_2 + i_4 = i_1 + i_5$$

$$\Rightarrow 5 = \frac{v_2 - v_1}{4} + \frac{v_2 - 0}{6}$$

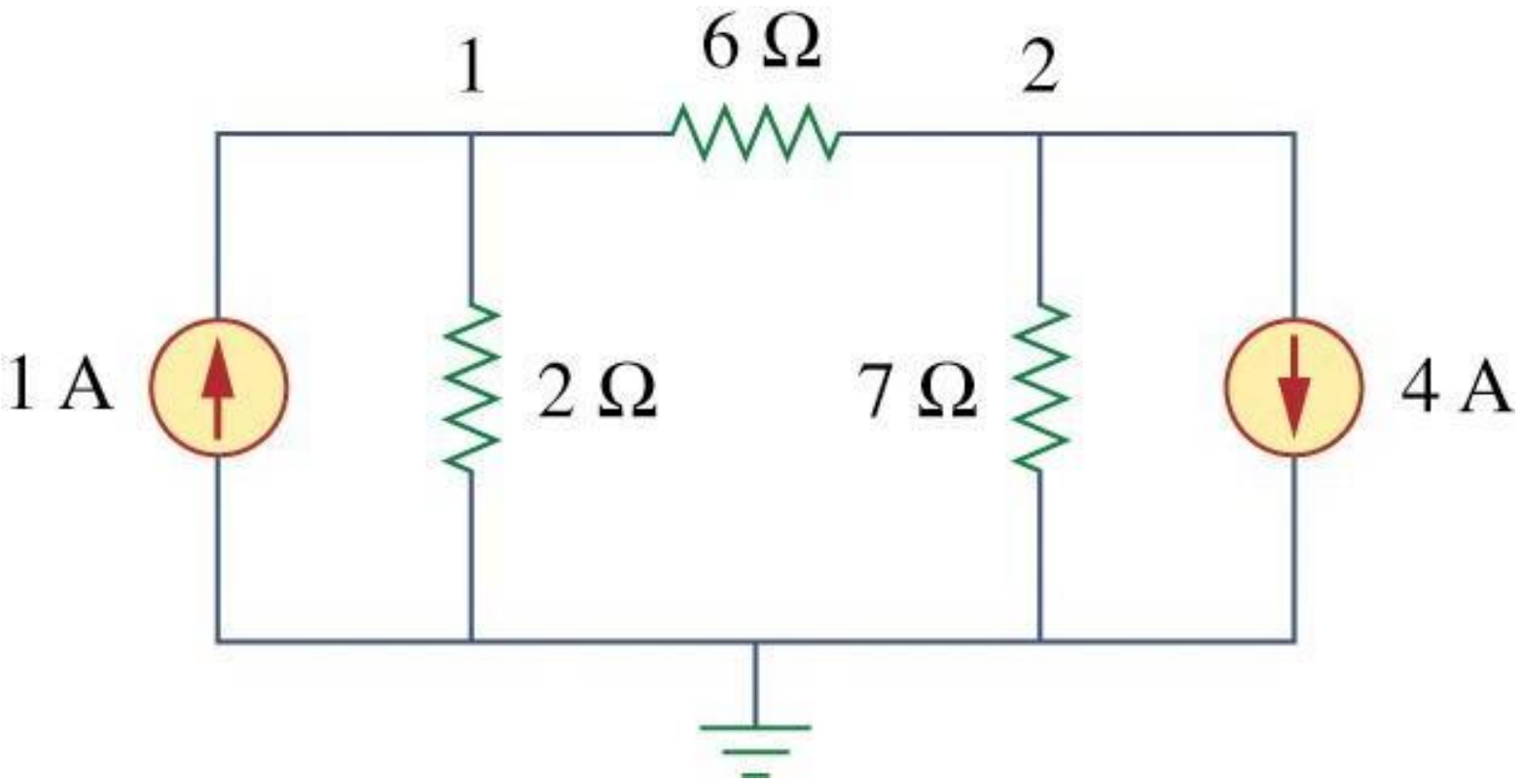


(b)

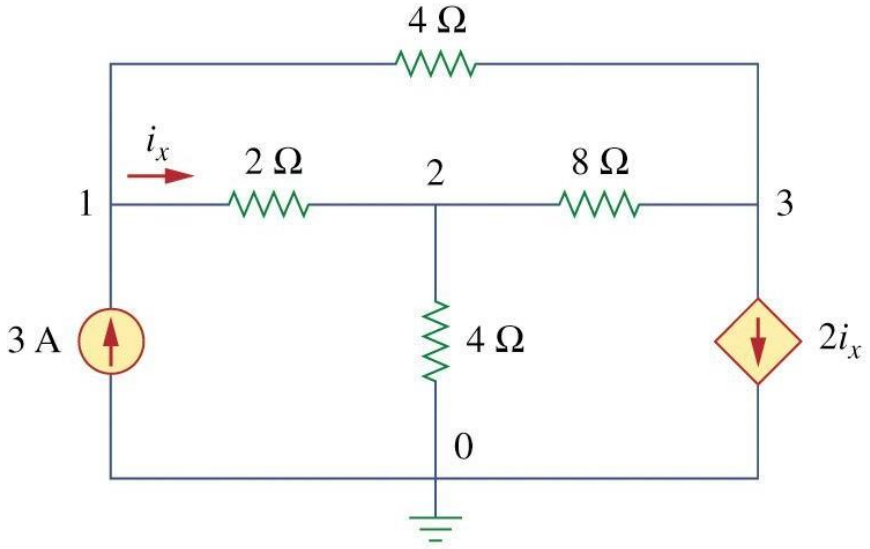
- In matrix form:

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{6}+\frac{1}{4} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

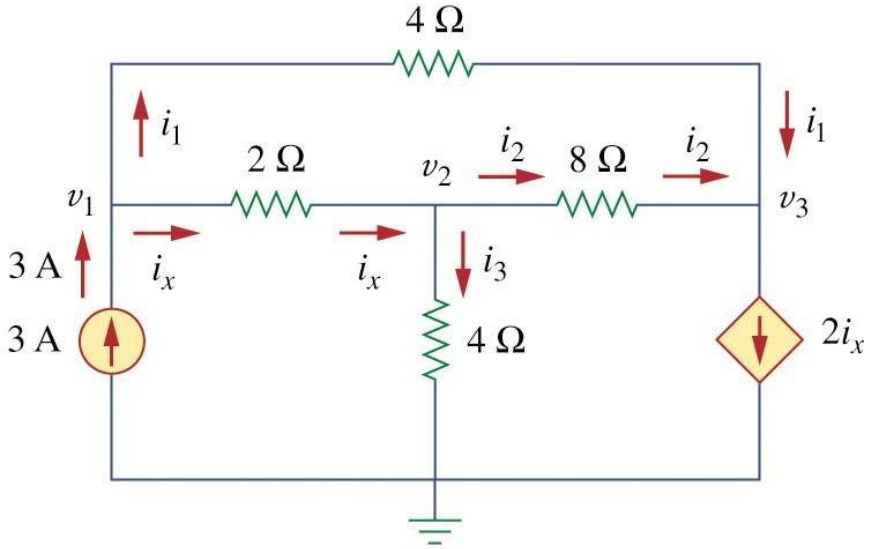
Practice



Determine the voltage at the nodes in Fig. below



(a)

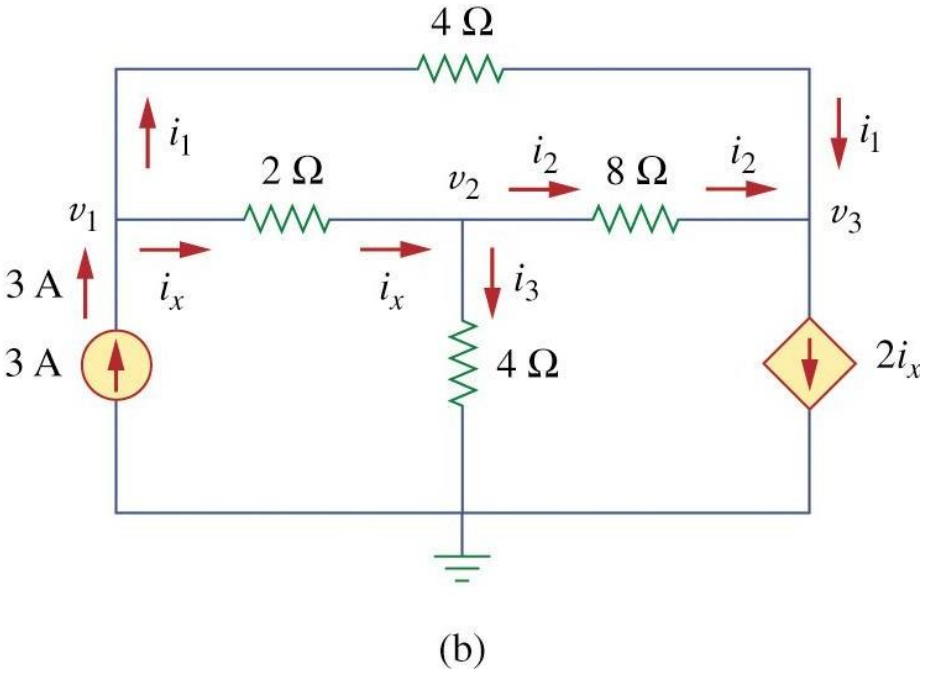


(b)

- At node 1,

$$3 = i_1 + i_x$$

$$\Rightarrow \quad 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

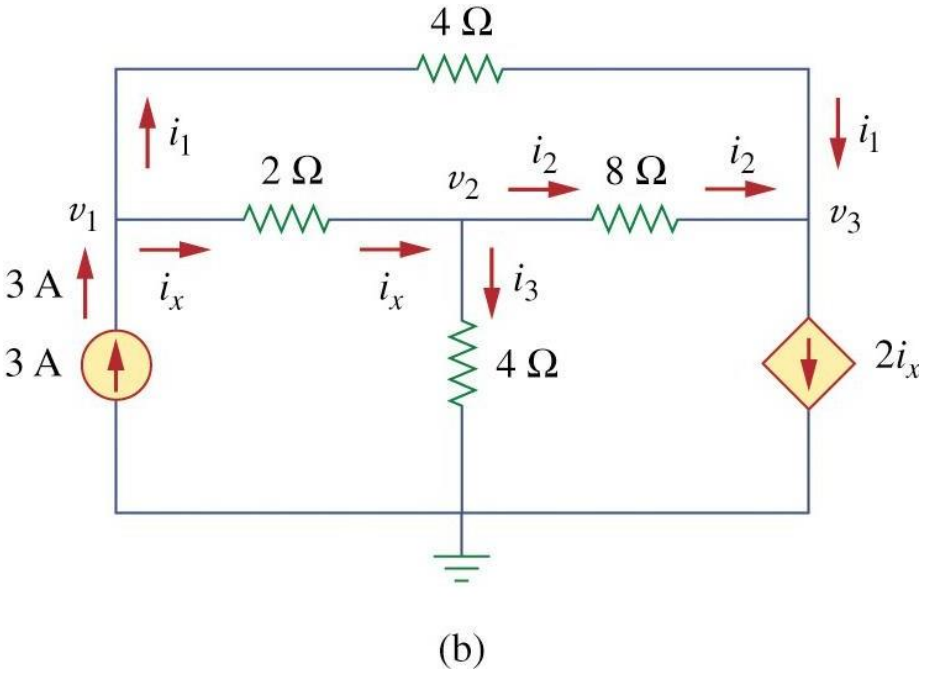


(b)

- At node 2

$$i_x = i_2 + i_3$$

$$\Rightarrow \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

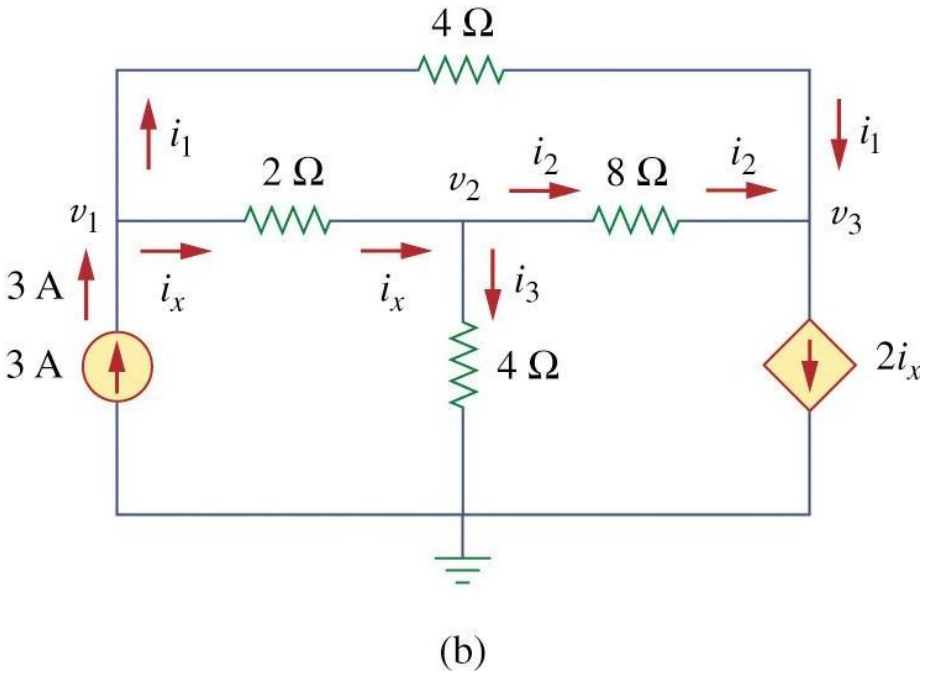


(b)

- At node 3

$$i_1 + i_2 = 2\,i_x$$

$$\Rightarrow \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_2 - v_1)}{2}$$



(b)

- In matrix form:

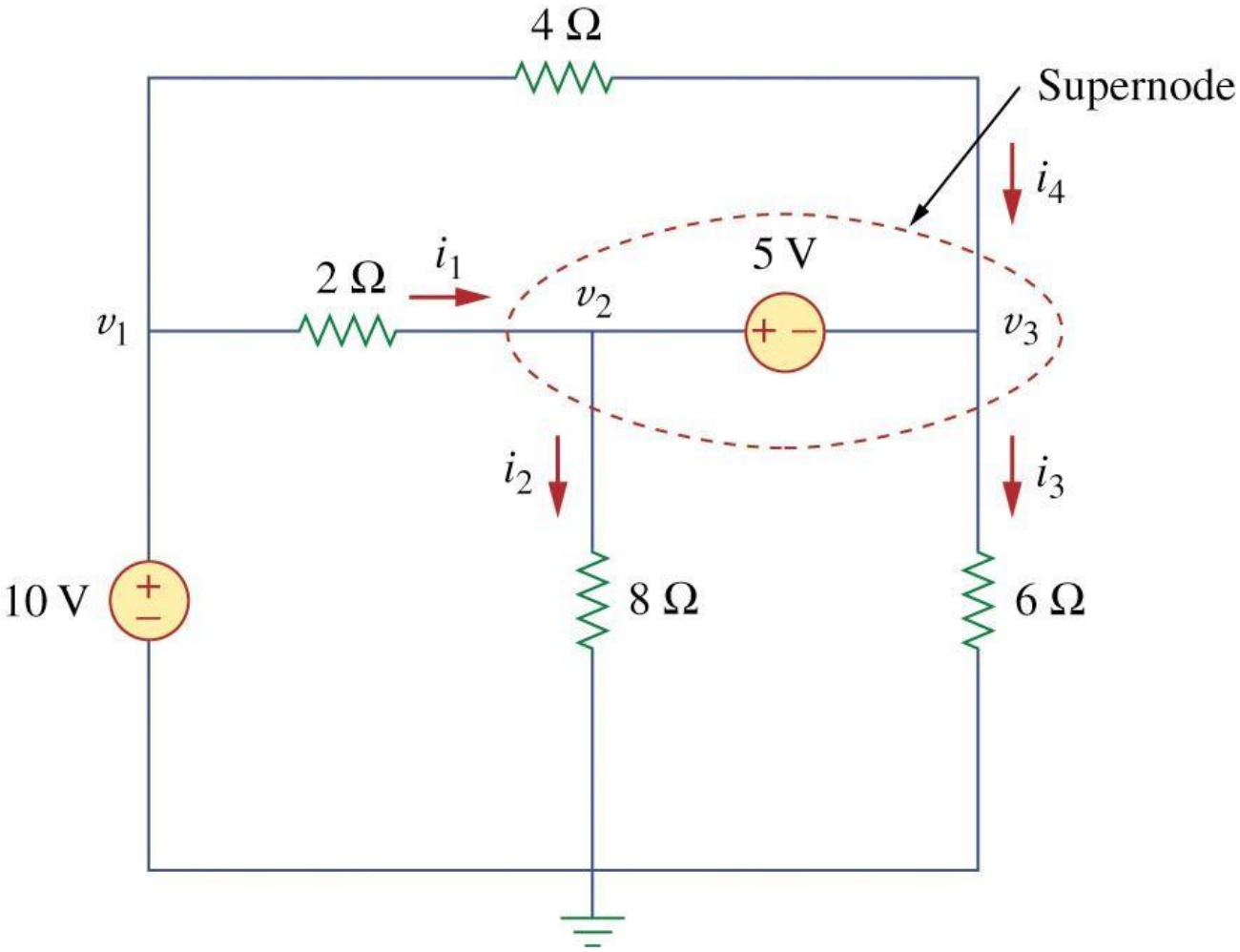
$$\begin{bmatrix} \frac{3}{4} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{7}{8} & -\frac{1}{8} \\ \frac{3}{4} & -\frac{9}{8} & \frac{3}{8} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

3.3 Nodal Analysis with Voltage Sources

- Case 1: The voltage source is connected between a nonreference node and the reference node: The nonreference node voltage is equal to the magnitude of voltage source and the number of unknown nonreference nodes is reduced by one.
- Case 2: The voltage source is connected between two nonreferenced nodes: a generalized node (supernode) is formed.

3.3 Nodal Analysis with Voltage Sources

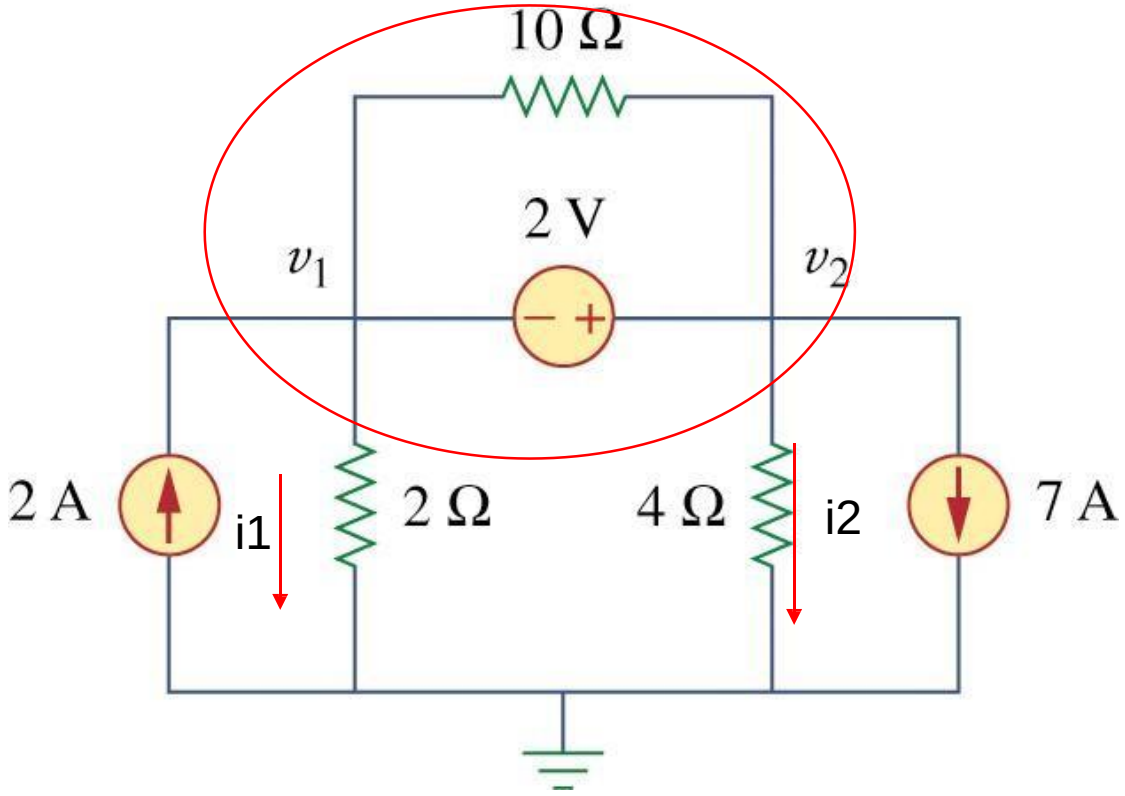
A circuit with a supernode.



- A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.
- The required two equations for regulating the two nonreference node voltages are obtained by the KCL of the supernode and the relationship of node voltages due to the voltage source.

Example 3.3

- For the circuit shown in Fig. 3.9, find the node voltages.

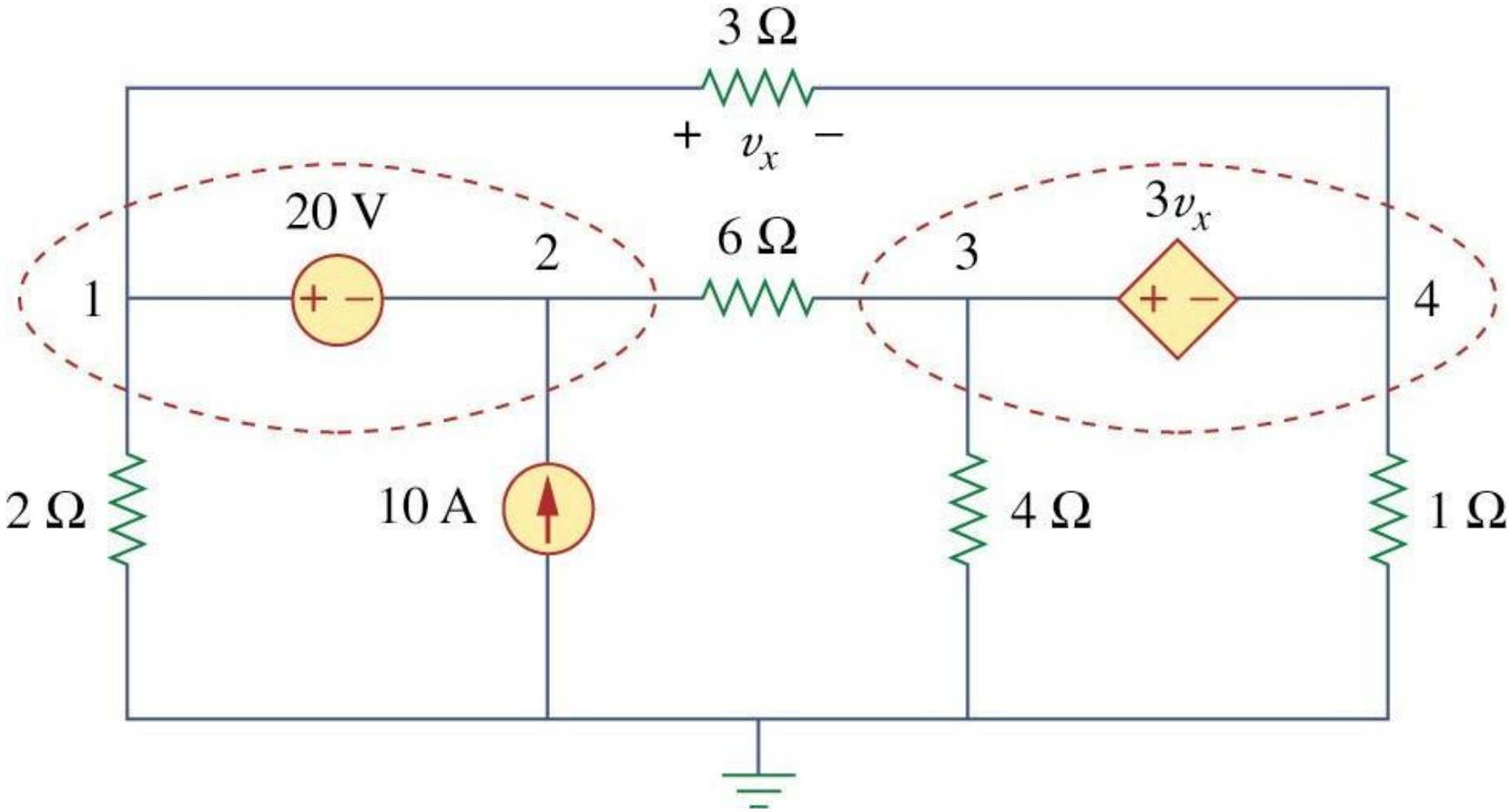


$$2 - 7 - i1 - i2 = 0$$

$$2 - 7 - \frac{v_1}{2} - \frac{v_2}{4} = 0$$

$$v_1 - v_2 = -2$$

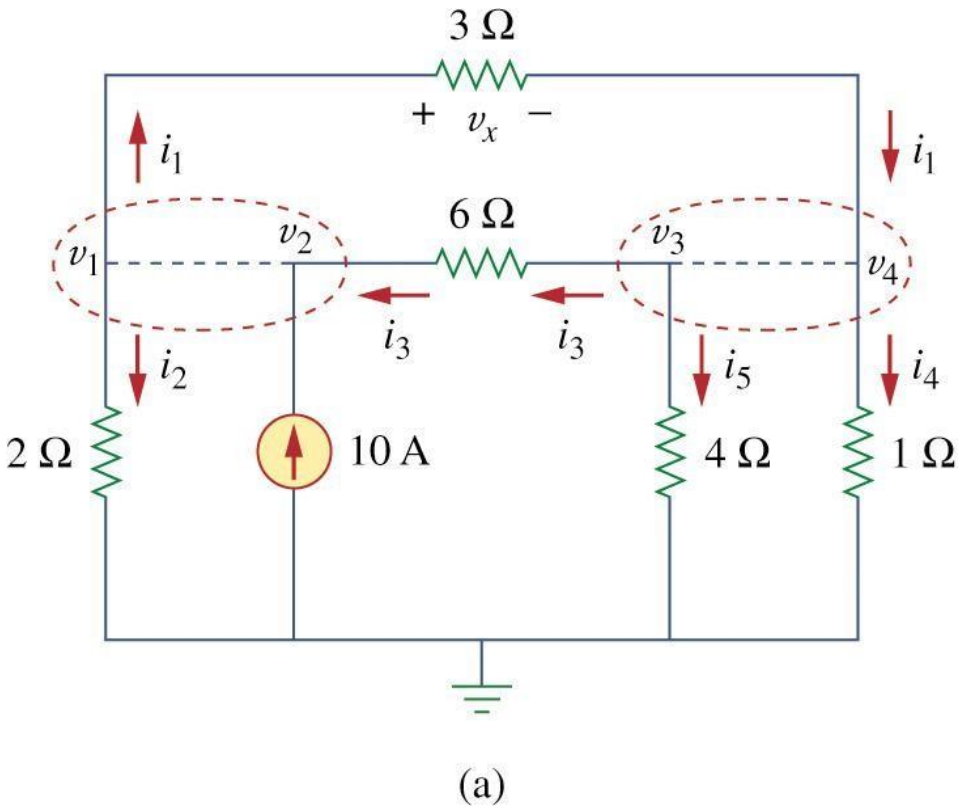
Find the node voltages in the circuit below.



- At suopernode 1-2,

$$\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}$$

$$v_1 - v_2 = 20$$

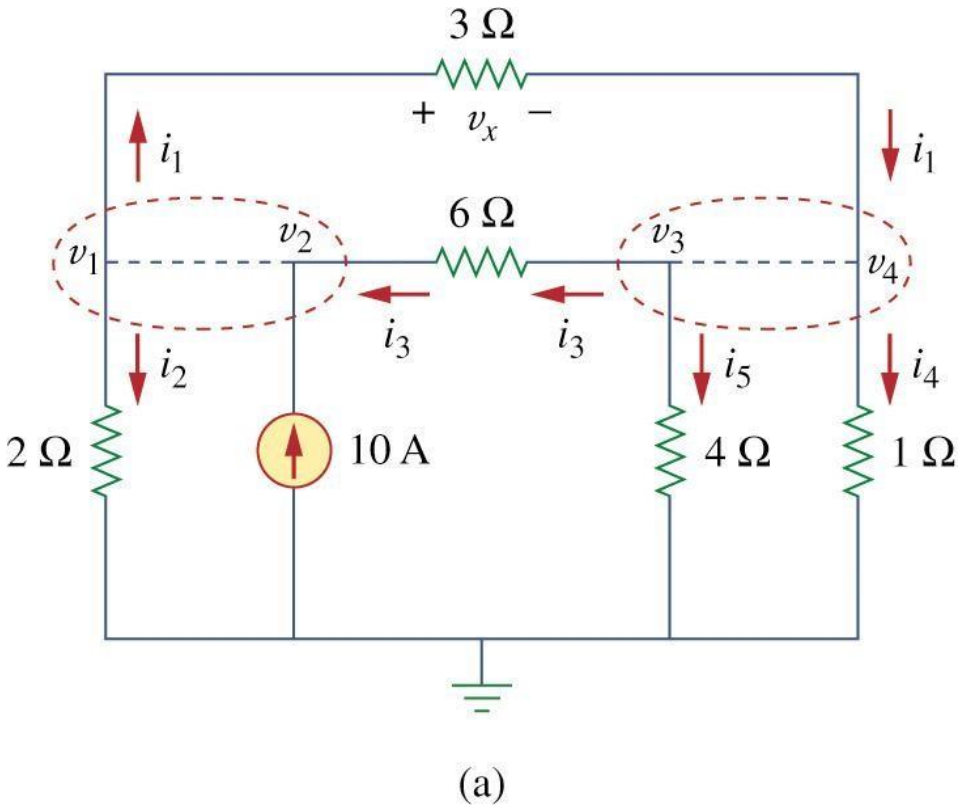


(a)

- At supernode 3-4,

$$\frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4}$$

$$v_3 - v_4 = 3(v_1 - v_4)$$

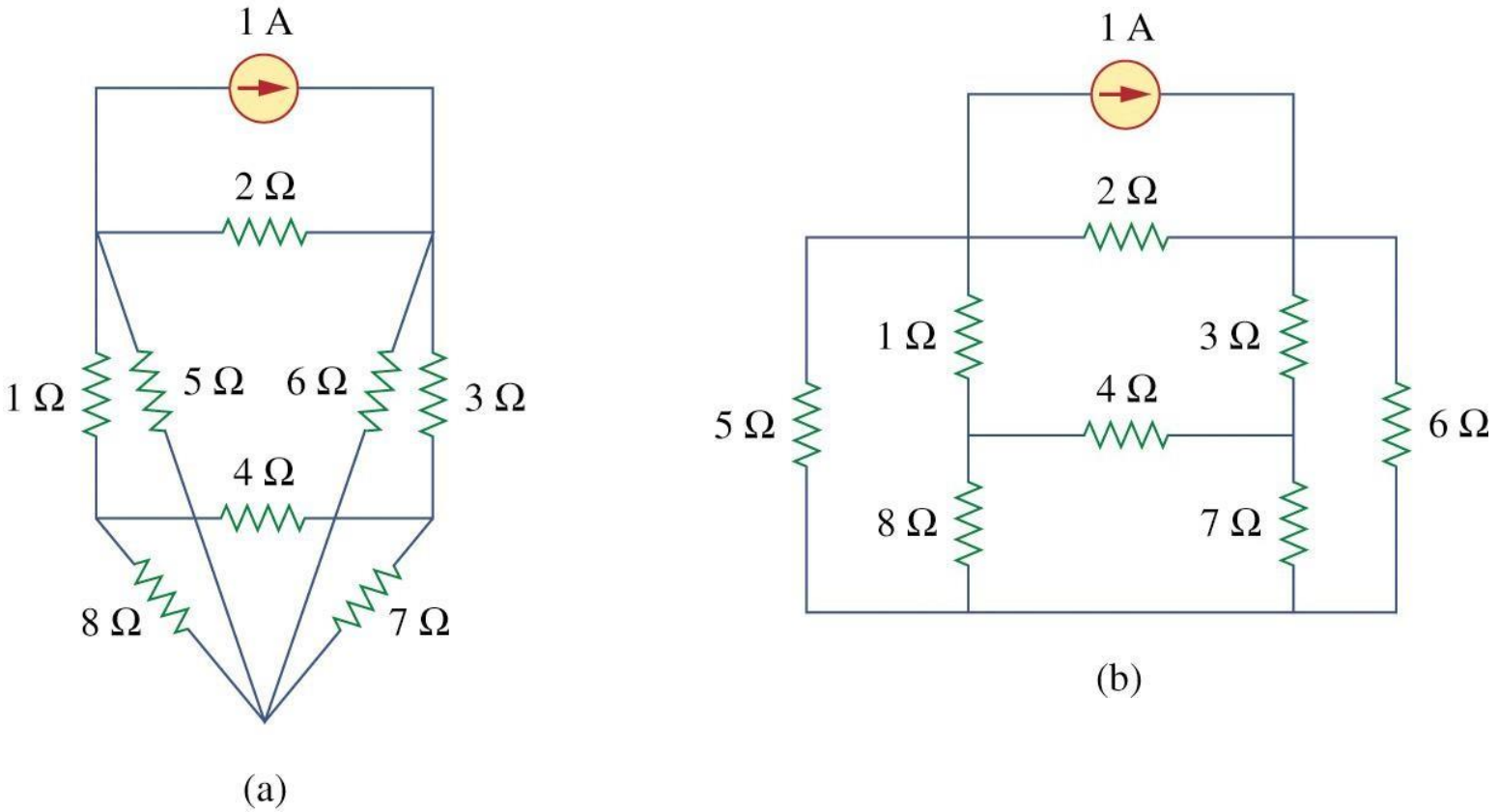


(a)

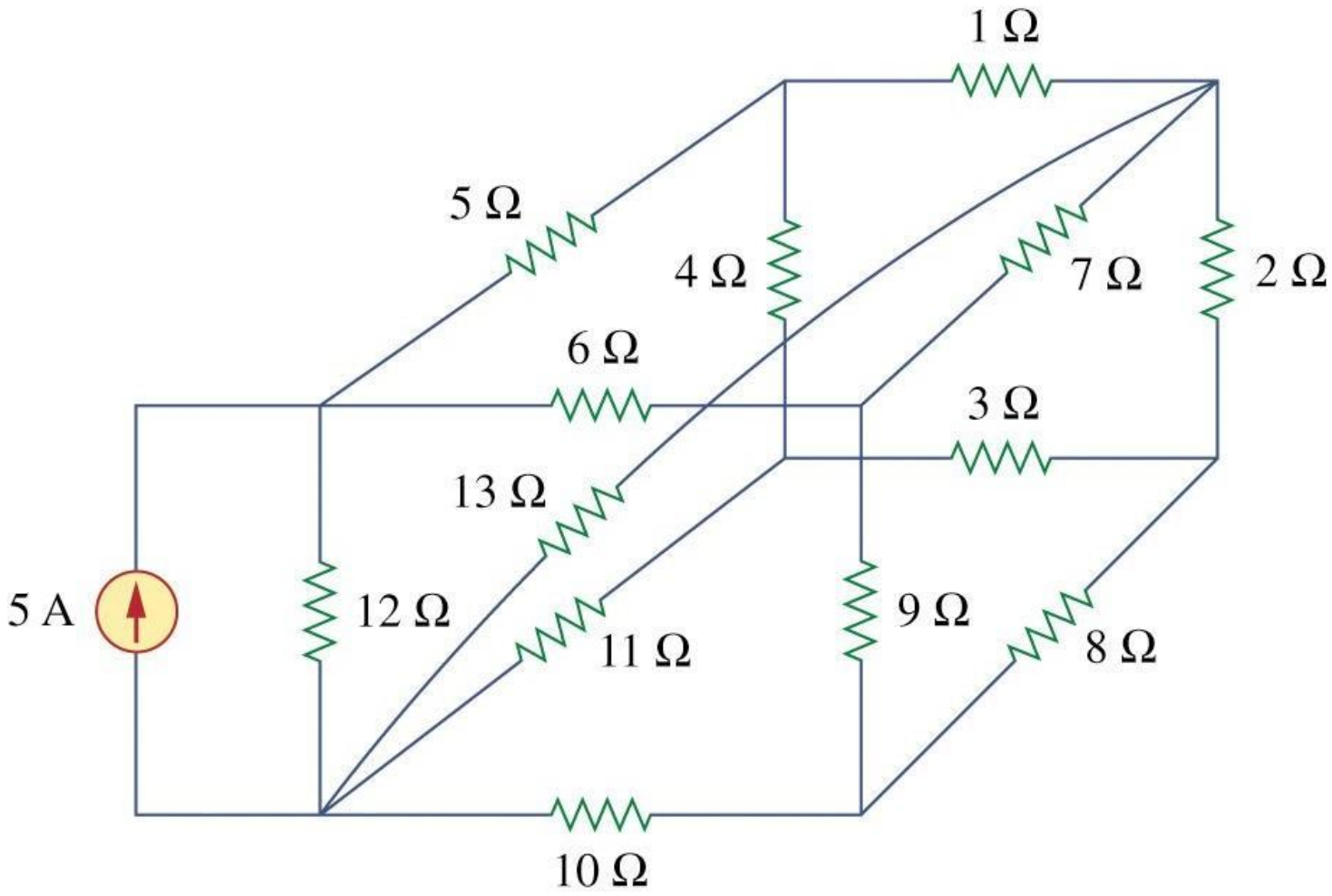
3.4 Mesh Analysis

- Mesh analysis: another procedure for analyzing circuits, applicable to planar circuit.
- A Mesh is a loop which does not contain any other loops within it

(a) A Planar circuit with crossing branches,
(b) The same circuit redrawn with no crossing branches.



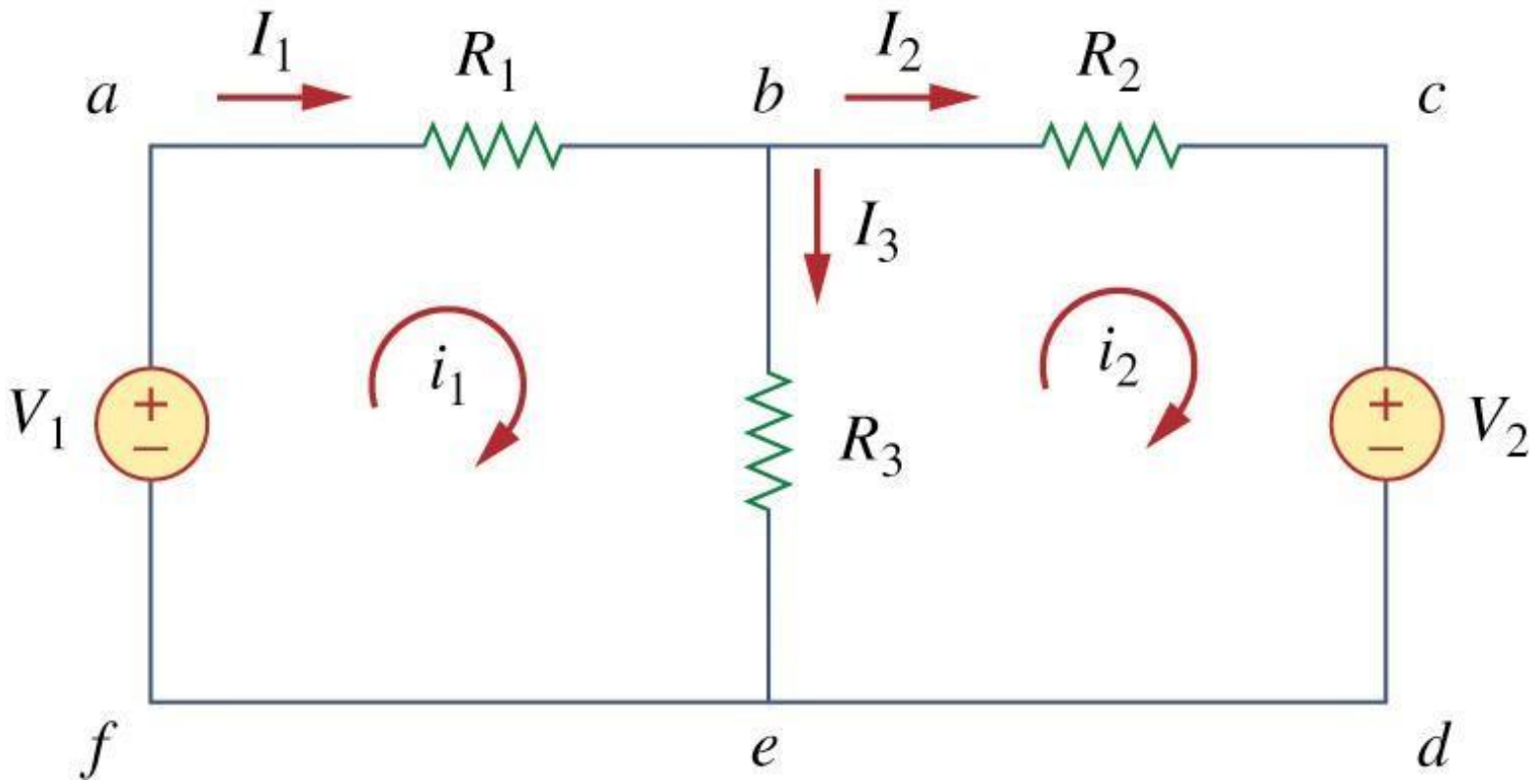
A nonplanar circuit.



- Steps to Determine Mesh Currents:
  1. Assign mesh currents i_1, i_2, .., i_n to the n meshes.
  2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
  3. Solve the resulting *n* simultaneous equations to get the mesh currents.

Fig. 3.17

A circuit with two meshes.



- Apply KVL to each mesh. For mesh 1,

$$-V_1 + R_1 i_1 + R_3 (i_1 - i_2) = 0$$

$$(R_1 + R_3) i_1 - R_3 i_2 = V_1$$

- For mesh 2,

$$R_2 i_2 + V_2 + R_3 (i_2 - i_1) = 0$$

$$-R_3 i_1 + (R_2 + R_3) i_2 = -V_2$$

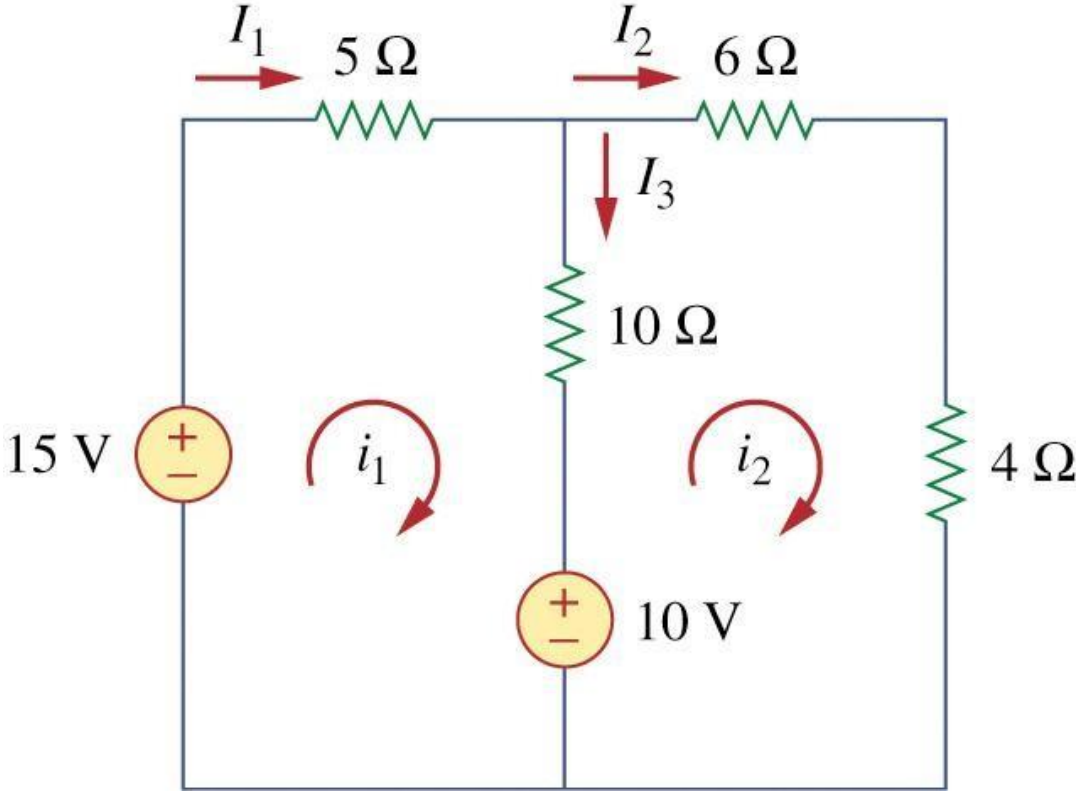
- Solve for the mesh currents.

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

- Use *i* for a mesh current and *I* for a branch current. It's evident from Fig. 3.17 that

$$I_1 = i_1, \quad I_2 = i_2, \quad I_3 = i_1 - i_2$$

- Find the branch current I_1, I_2, and I_3 using mesh analysis.



- For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

$$\boxed{3i_1 - 2i_2 = 1}$$

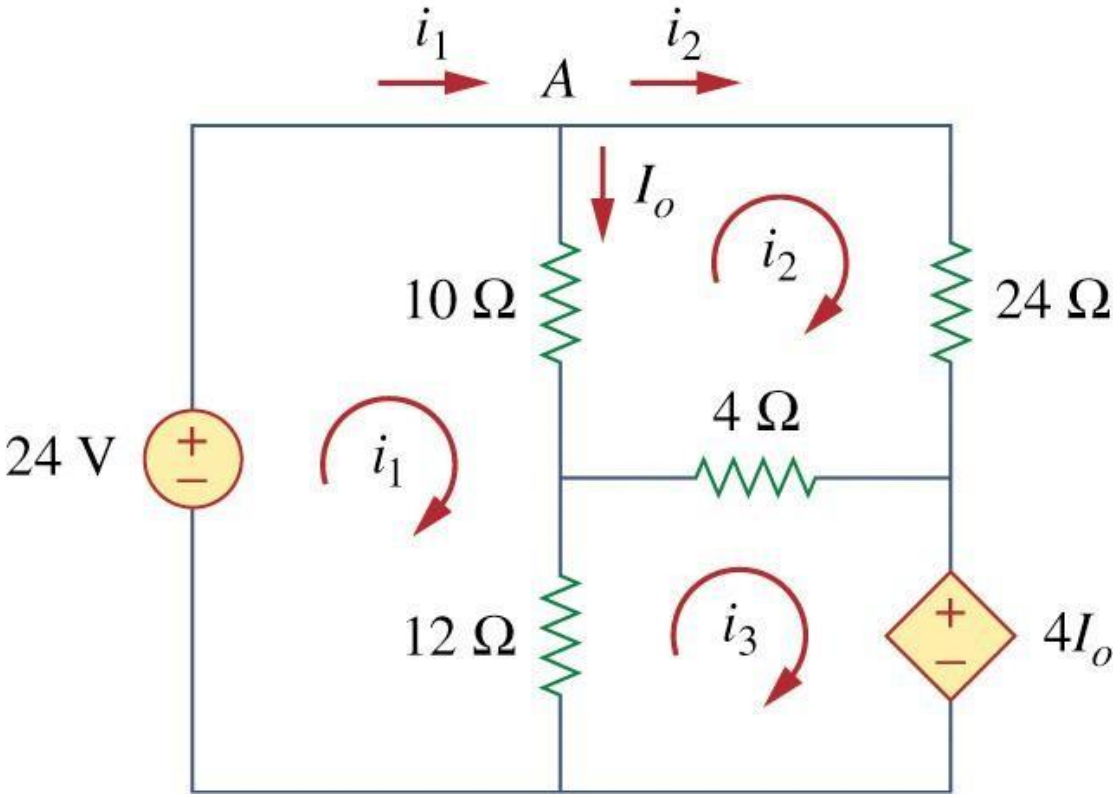
- For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

$$\boxed{i_1 = 2i_2 - 1}$$

- We can find i_1 and i_2 by substitution method or Cramer's rule. Then, $I_1 = i_1, \quad I_2 = i_2, \quad I_3 = i_1 - i_2$

- Use mesh analysis to find the current I_0 in the circuit.



- Apply KVL to each mesh. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

$$11 i_1 - 5 i_2 - 6 i_3 = 12$$

- For mesh 2,

$$24 i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

$$-5 i_1 + 19 i_2 - 2 i_3 = 0$$

- For mesh 3, $4I_0 + 12\,(i_3 - i_1) + 4(i_3 - i_2) = 0$

$$\text{At node A, } I_0 = I_1 - i_2,$$

$$4\,(i_1 - i_2) + 12\,(i_3 - i_1) + 4\,(i_3 - i_2) = 0$$

$$-i_1 - i_2 + 2i_3 = 0$$

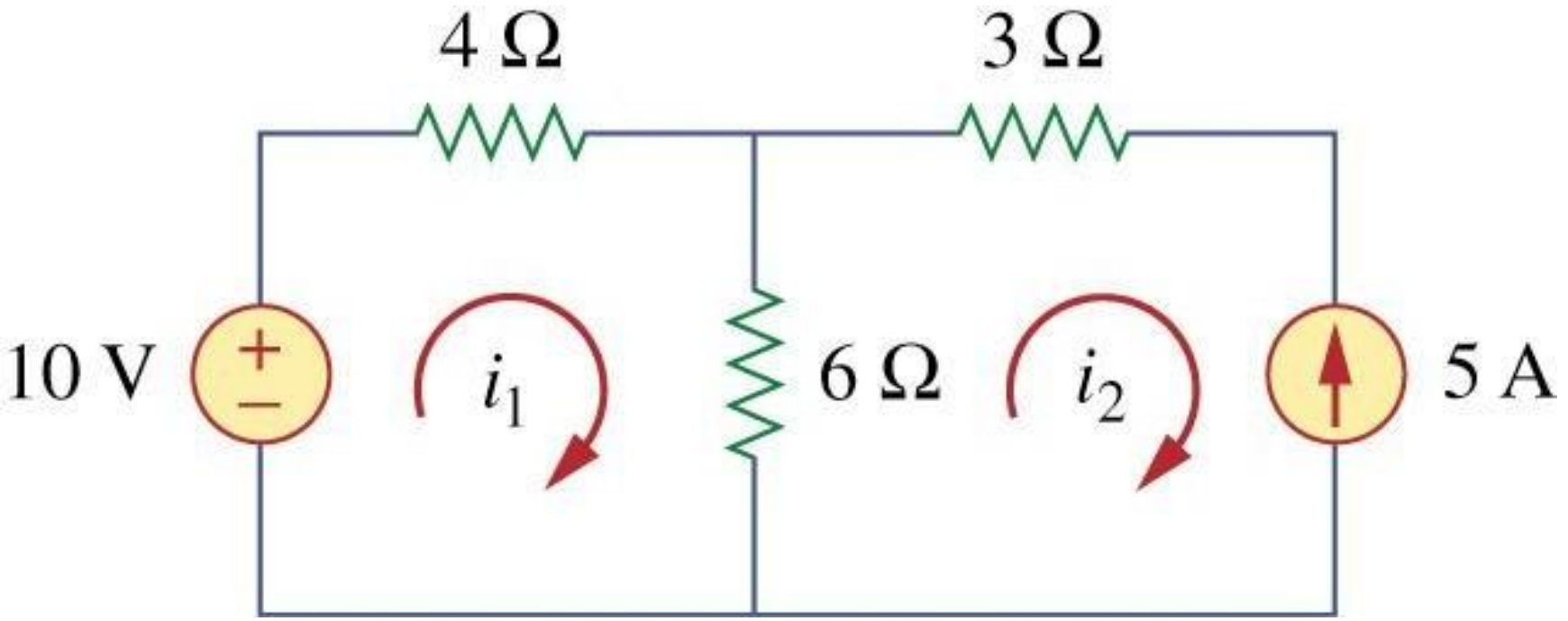
- In matrix from become

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

we can calculus i_1, i_2 and i_3 by Cramer's rule, and find I_0.

3.5 Mesh Analysis with Current Sources

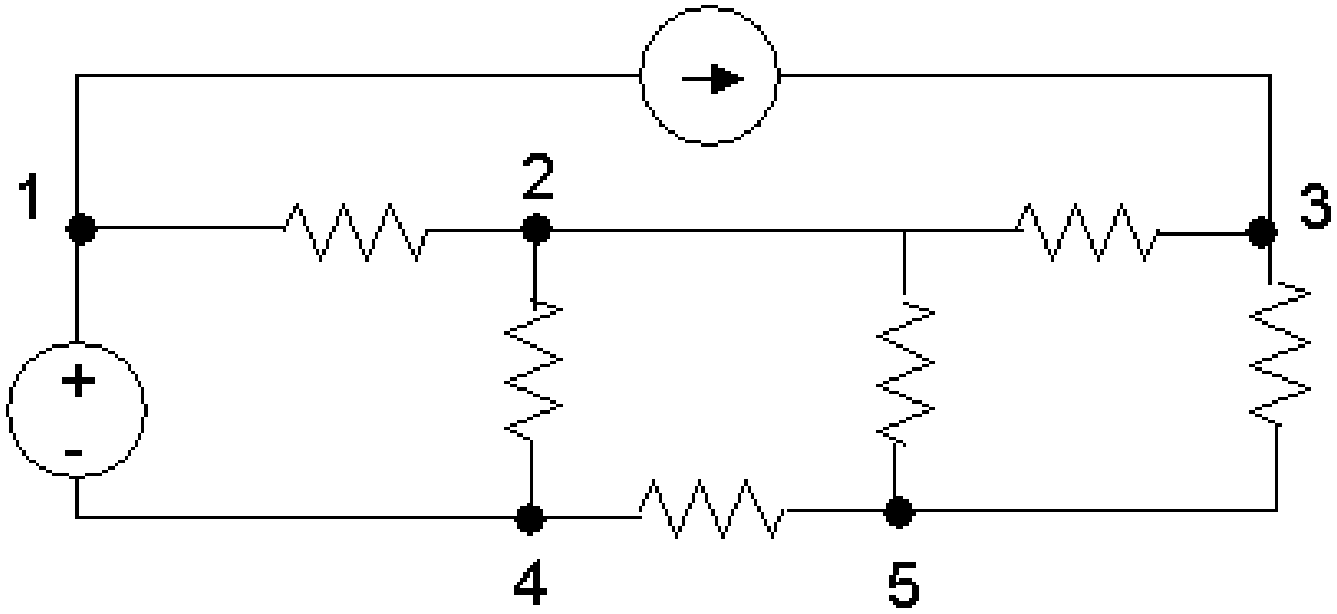
A circuit with a current source.



NETWORK TOPOLOGY

Graph Theory in Circuit Analysis

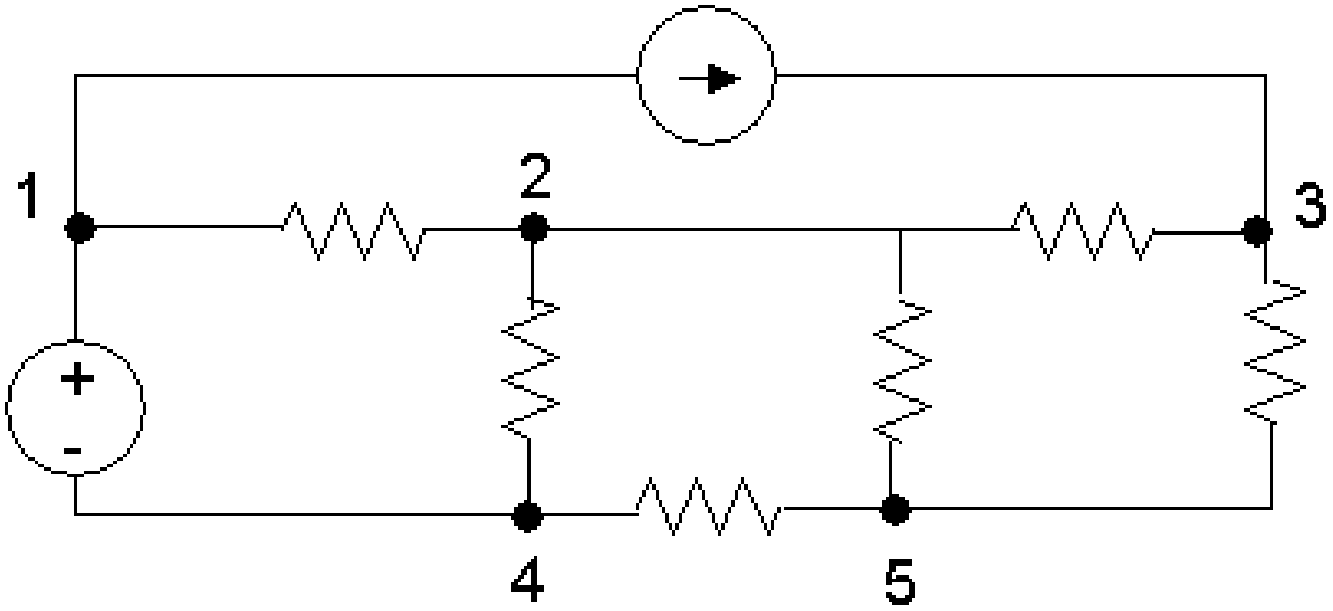
Suppose we wish to find the node voltages of the circuit below.



We know how to do this by hand.

For large-scale circuits, we may wish to do this via a computer simulation (i.e. PSpice). We will need to express this circuit in a standard form for input to the program.

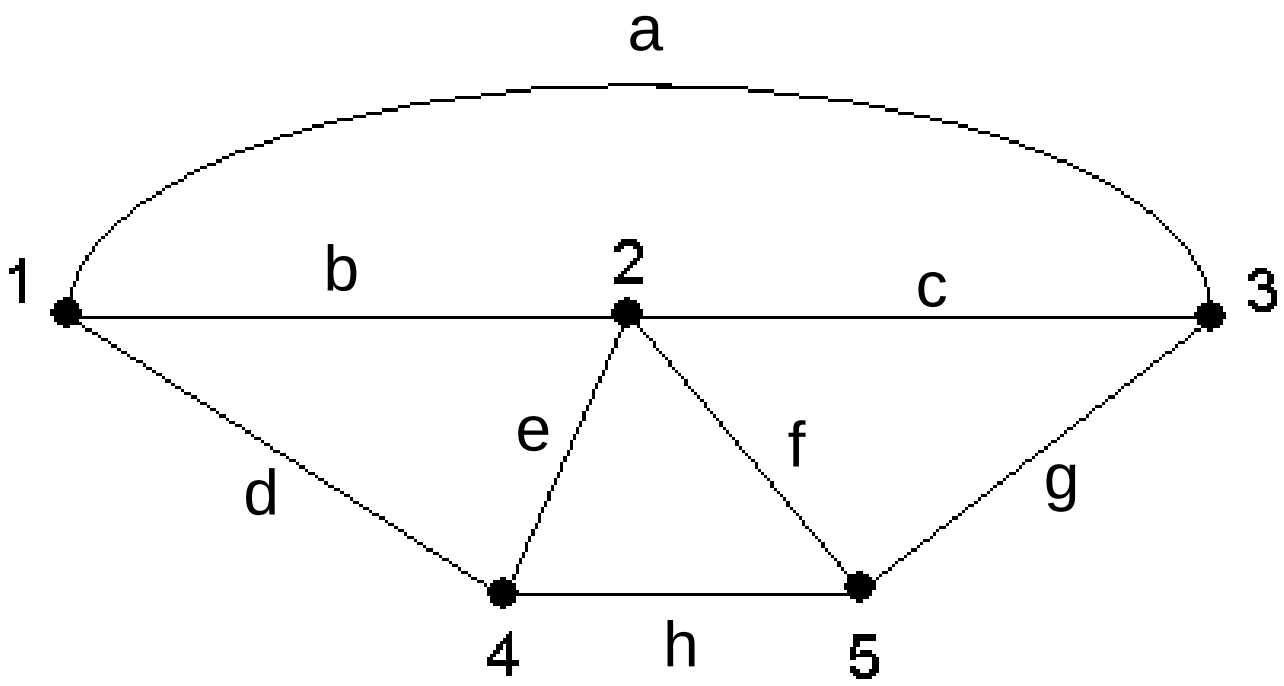
Graph Theory in Circuit Analysis



Whether the circuit is input via a GUI or as a text file, at some level the circuit will be represented as a graph, with elements as edges and nodes as nodes.

For example, when entering a circuit into PSpice via a text file, we number each node, and specify each element (edge) in the circuit with its value and endpoints.

Graph of a Circuit

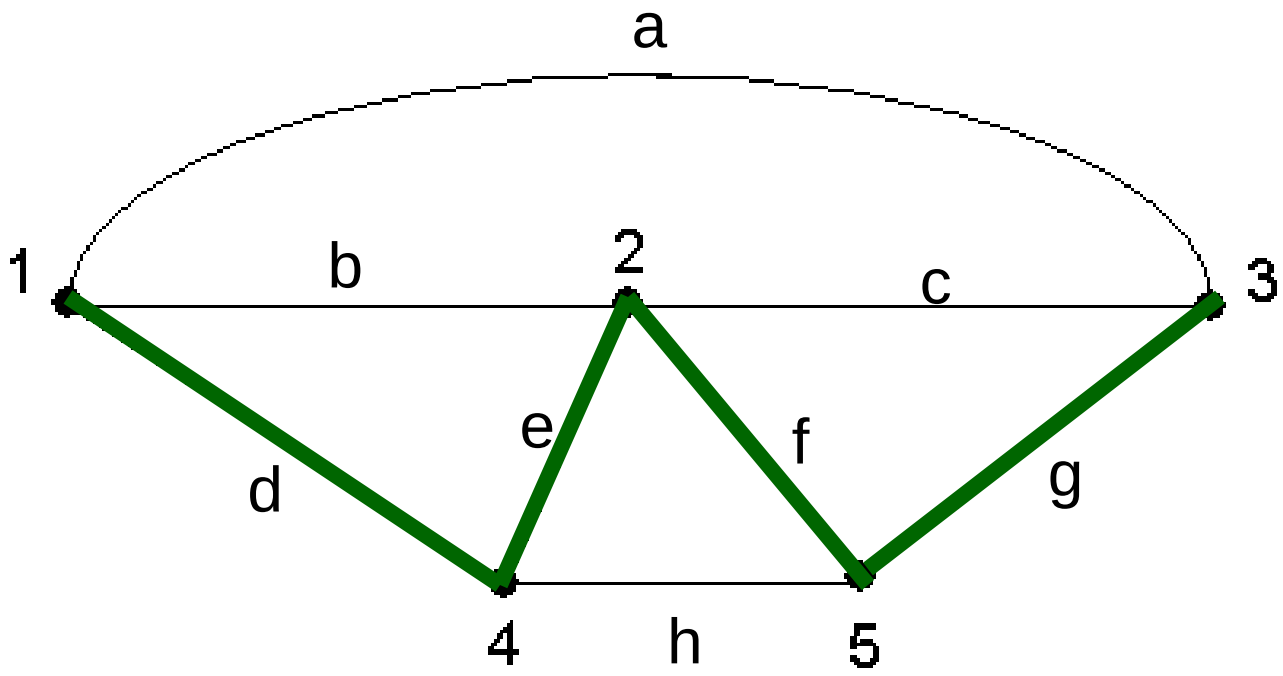


Here is a **graph** of the circuit. It is simply the circuit without elements.

We refer to the lines above as **edges** (and the nodes are **nodes**).

The graph provides **connectivity** information. To actually solve the circuit using this graph, the types of elements forming the edges would need to be provided.

Trees and Co-Trees

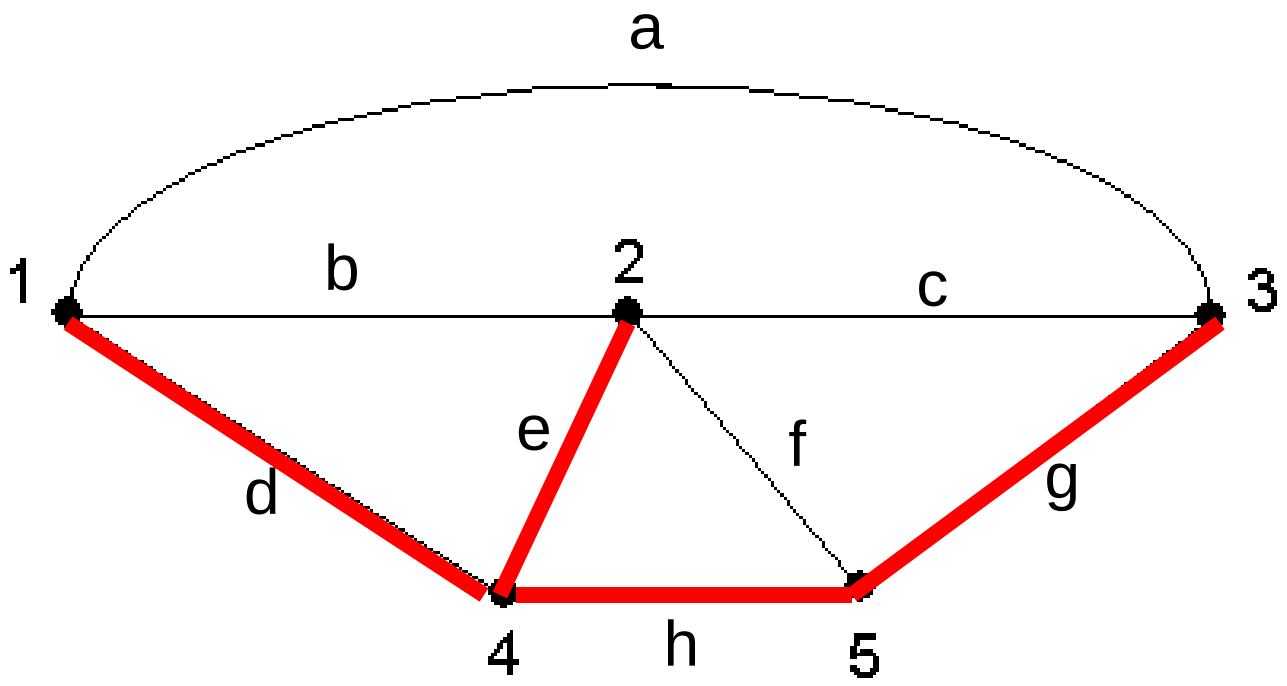


A **tree** is defined as any set of edges in a graph that touches every node *without forming any closed paths.*

Also known as Hamiltonian path!

Each tree has a **co-tree**, which is the set of edges not in the tree.

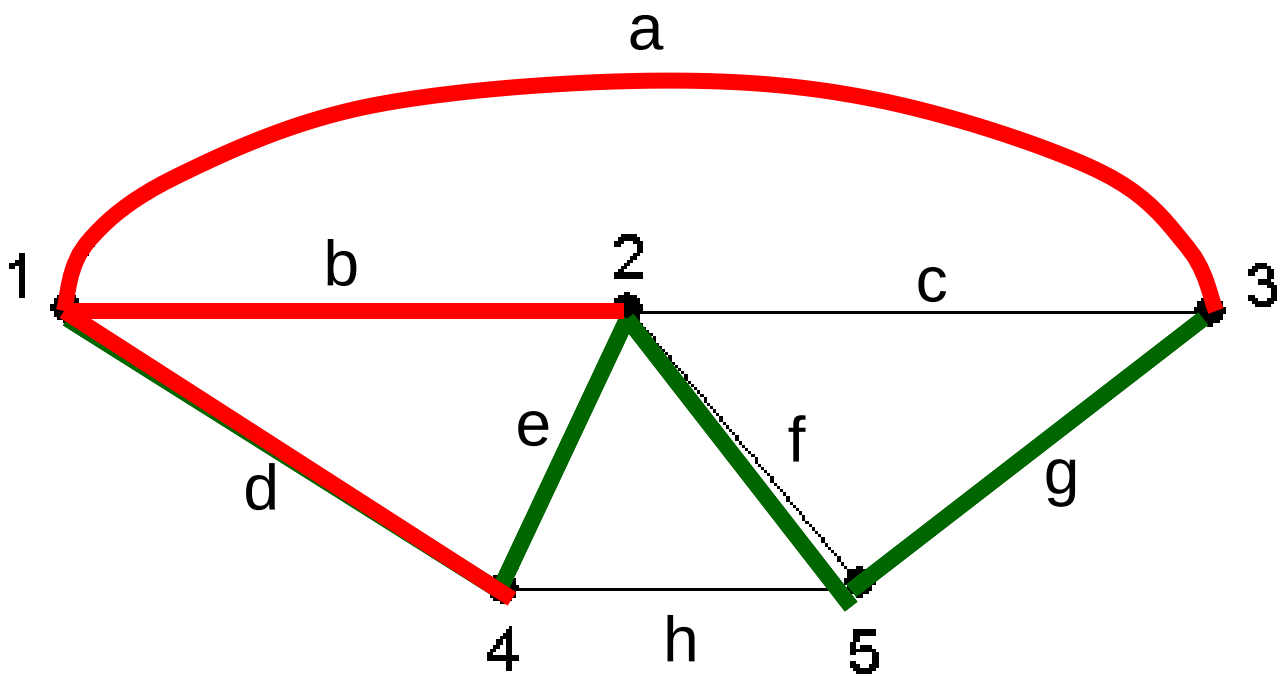
Cut Set



A **cut set** is a *minimal* set of edges that, when broken, breaks the graph into two completely separate parts (two groups of nodes.

Minimal means that a cut set cannot contain another smaller cut set that would break the graph into the same two parts.

Fundamental Cut Set



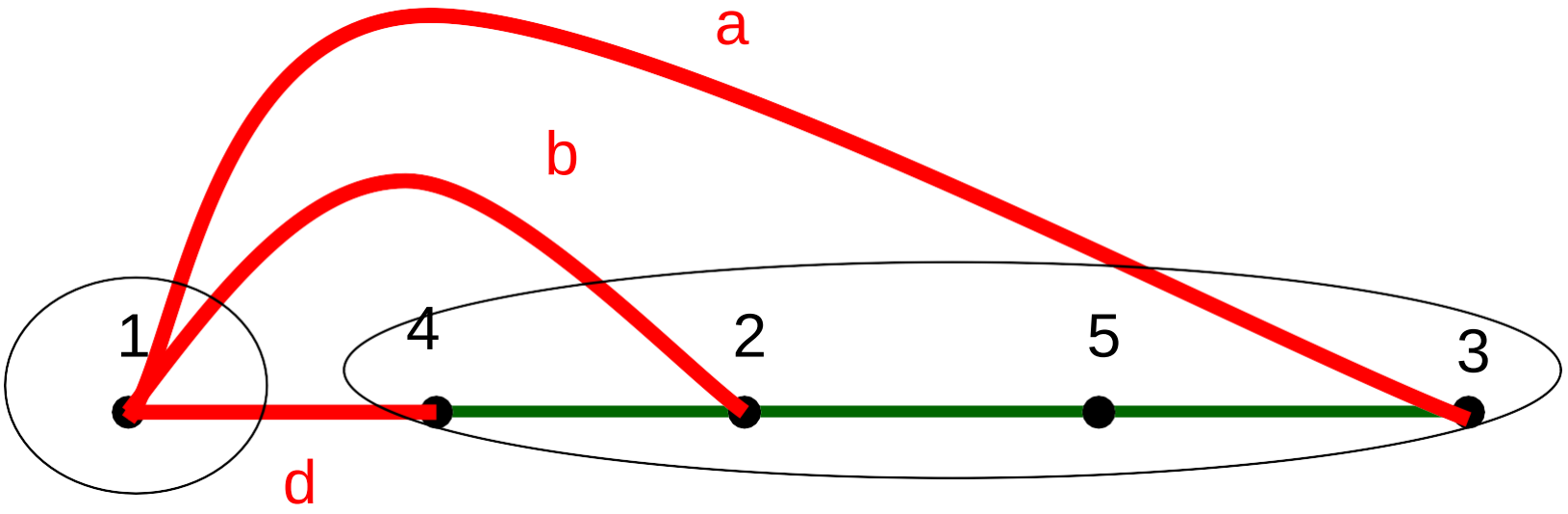
Suppose I am given a tree.

A **fundamental cut set** w.r.t. that tree is a cut set that only contains one branch of the tree.

There may be many fundamental cut sets w.r.t. a given tree.

Finding Fundamental Cut Sets Systematically

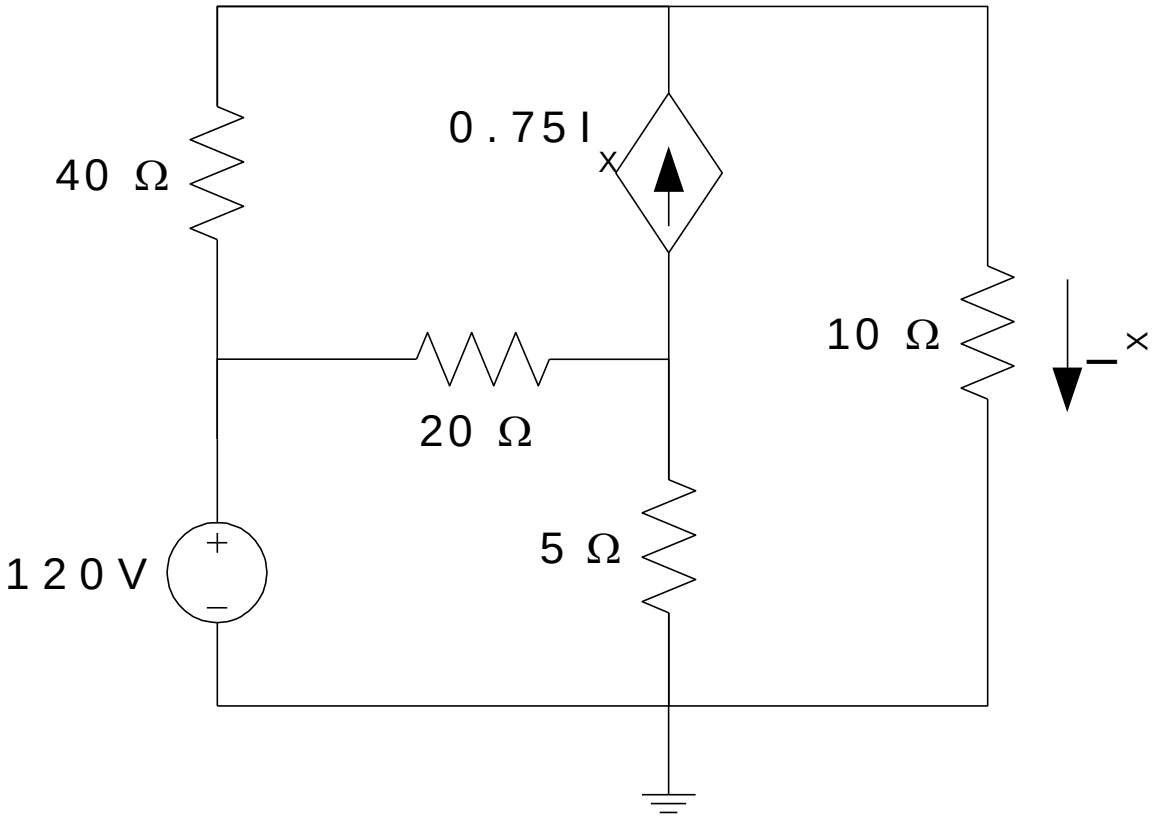
1. Redraw the graph with the tree in a straight line.
2. For each tree edge, form its fundamental cut set as follows:
   2a) that tree edge is a member of this fundamental cut set
   2b) cut that edge…what two groups of nodes are separated?
   2c) the fundamental cut set also contains all edges in the co-tree that connect these two groups.



Notes

- All of this can be done computationally.
  - Graph algorithms
  - Linear equation solution
- This algorithm shows why nodal analysis always works: you get *n-1* independent linear equations in *n-1* unknowns.
- The fundamental cut sets ensure independence of the equations—unless the circuit has impossible elements.
  - Each fundamental cut set contains a unique element (edge) from the tree. So each KCL equation provides new info.
  - The elements themselves could destroy the independence (redundant dependent source, shorted voltage source…) but this won‘t happen in real life circuits.

Example



Find the node voltages using the graph method.

Circuit from Nilsson‘s *Electric Circuits*,Addison-Wesley, 1993.

UNIT – III

SINGLE PHASE AC CIRCUITS AND ANALYSIS

Objectives: After completing this module, you should be able to:

- Describe the sinusoidal variation in ac current and voltage, and calculate their effective values.
- Write and apply equations for calculating the inductive and capacitive reactances for inductors and capacitors in an ac circuit.
- Describe, with diagrams and equations, the phase relationships for circuits containing resistance, capacitance, and inductance.

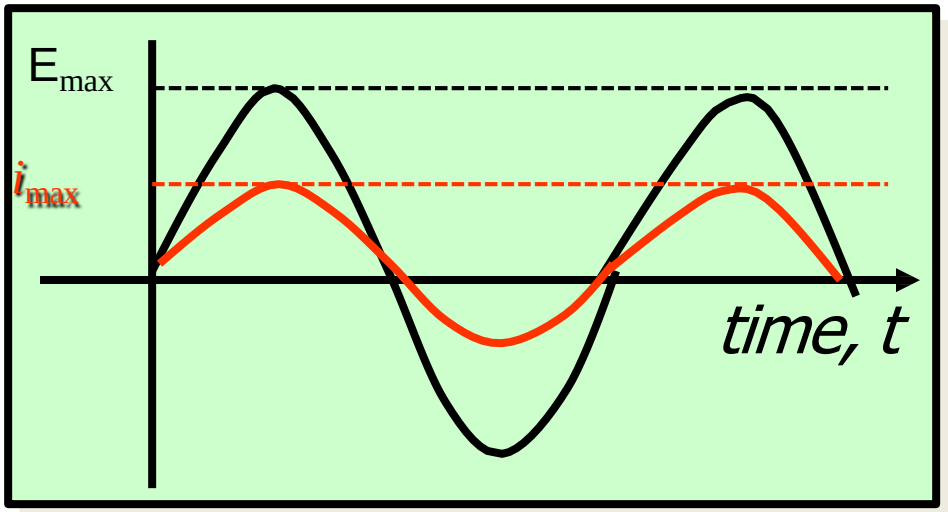
Alternating Currents

An alternating current such as that produced by a generator has no direction in the sense that direct current has. The magnitudes vary sinusoidally with time as given by:

AC-voltage and current

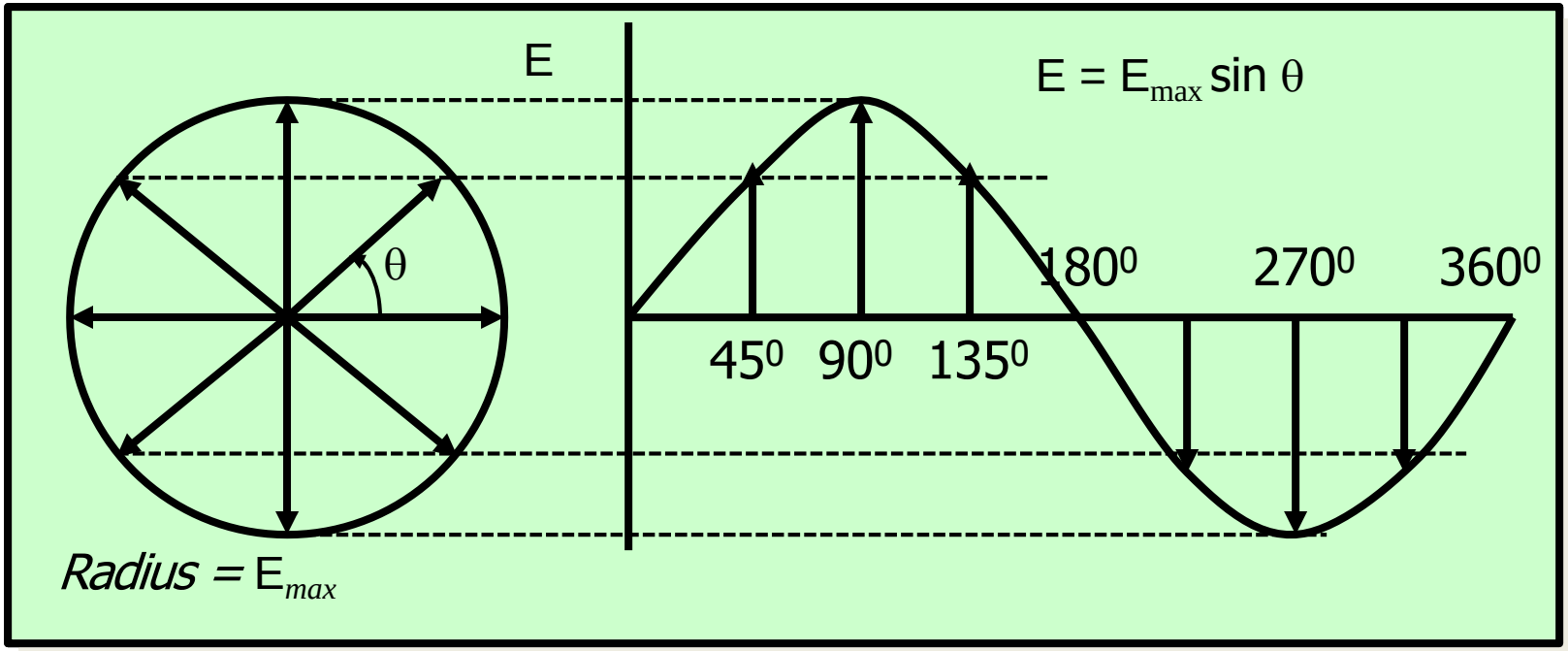
$E = E_{max} \sin \theta$

$i = i_{max} \sin \theta$



Rotating Vector Description

The coordinate of the emf at any instant is the value of $E_{max} \sin \theta$.
Observe for incremental angles in steps of 45^0. Same is true for *i*.



Example 1: For a particular device, the house ac voltage is 120-V and the ac current is 10 A. What are their maximum values?

$i_{eff} = 0.707\ i_{max}$

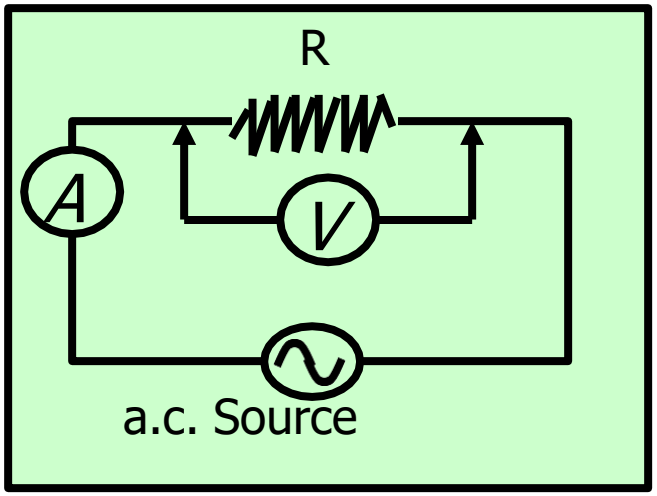
$V_{eff} = 0.707\ V_{max}$

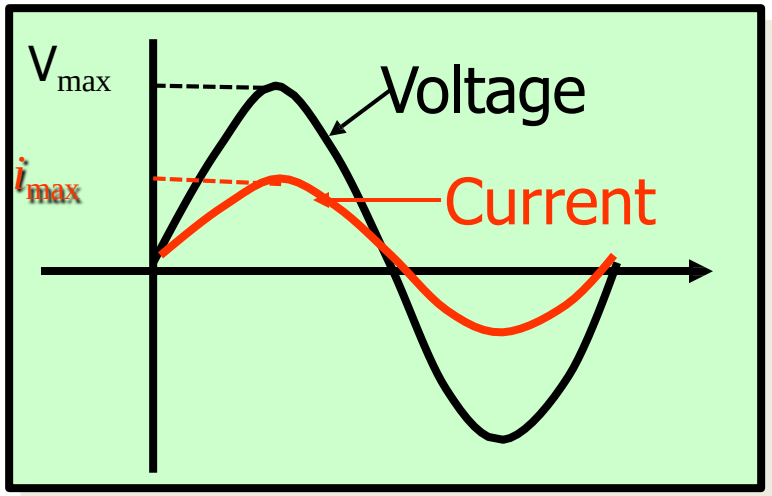
$i_{max} = 14.14$ A

$V_{max} = 170$ V

The ac voltage actually varies from +170 V to -170 V and the current from 14.1 A to –14.1 A.

Pure Resistance in AC Circuits

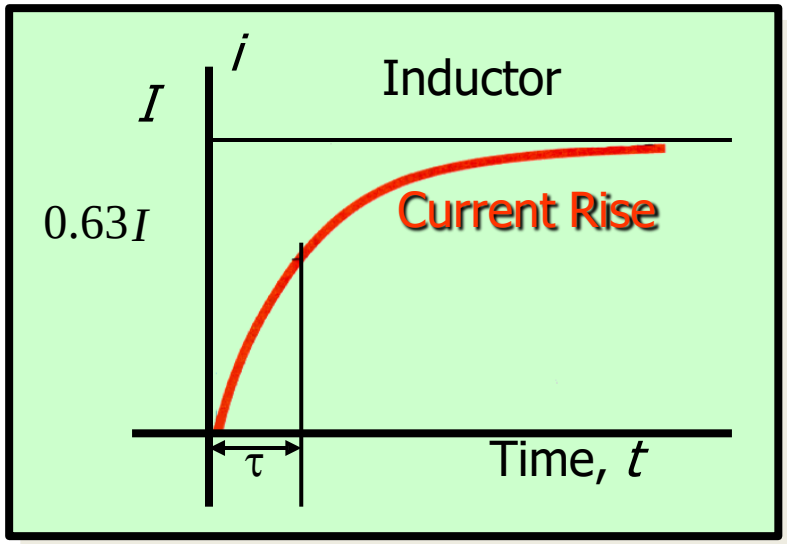


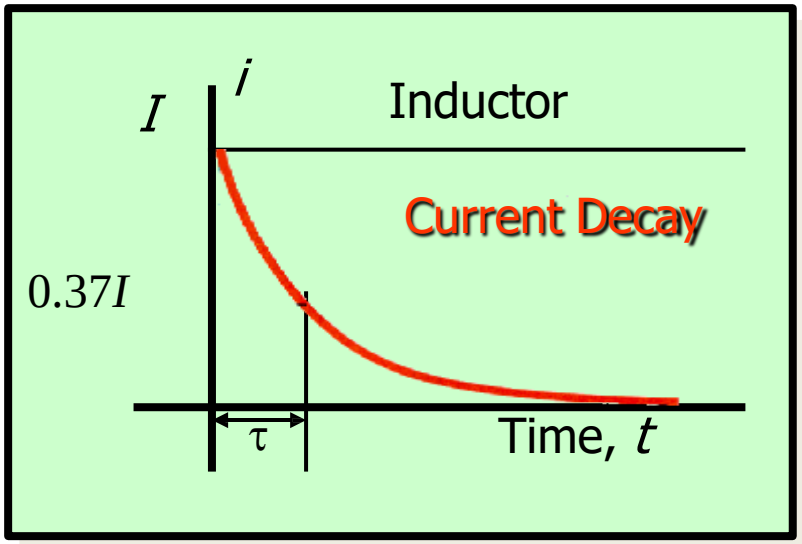


Voltage and current are in phase, and Ohm's law applies for effective currents and voltages.

Ohm's law: $V_{eff} = i_{eff}R$

AC and Inductors

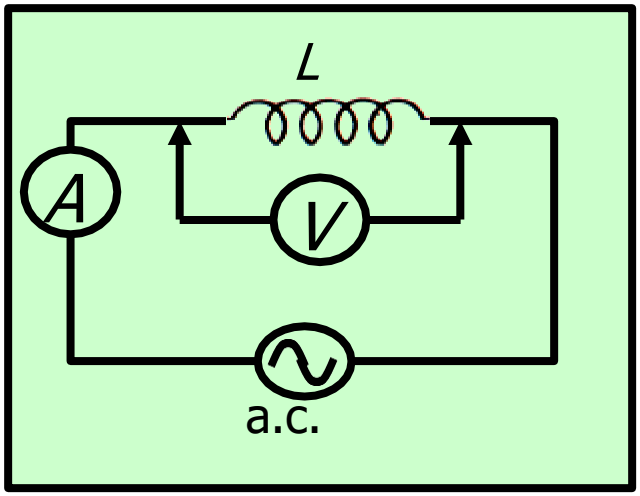


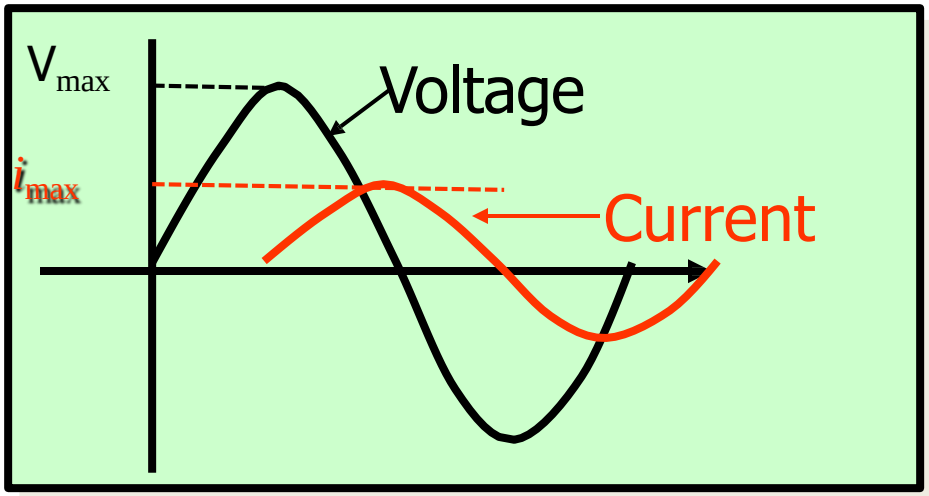


The voltage *V* peaks first, causing a rapid rise in *i* current which then peaks as the emf goes to zero. Voltage leads (peaks before) the current by 90°.

Voltage and current are out of phase.

A Pure Inductor in AC Circuit



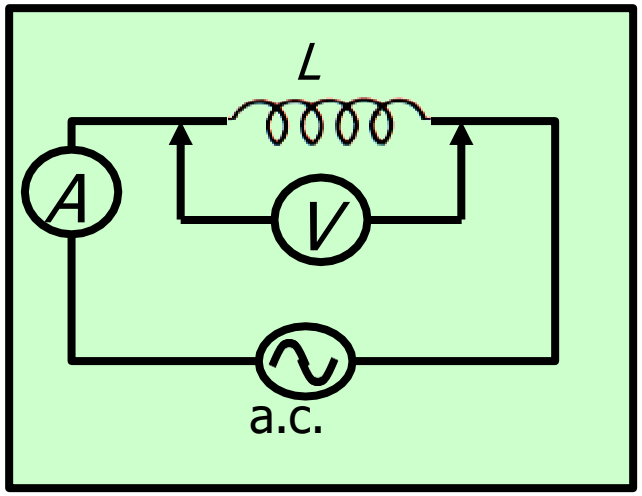


The voltage peaks 90^0 before the current peaks. One builds as the other falls and vice versa.

The reactance may be defined as the non resistive opposition to the flow of ac current.

Inductive Reactance

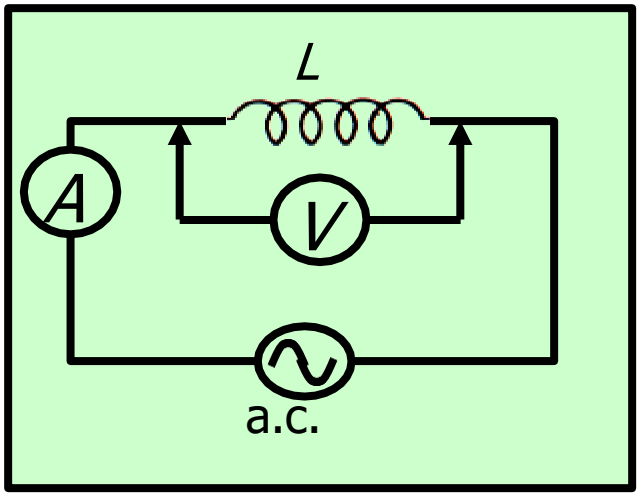
The back emf induced by a changing current provides opposition to current, called inductive reactance X_L.



Such losses are temporary, however, since the current changes direction, periodically re-supplying energy so that no net power is lost in one cycle.

Inductive reactance X_L is a function of both the inductance and the frequency of the ac current.

Calculating Inductive Reactance



Inductive Reactance:

$$X_L = 2\pi fL \quad \text{Unit is the } \Omega$$

$$\text{Ohm's law: } V_L = i X_L$$

The voltage reading V in the above circuit at the instant the ac current is i can be found from the inductance in H and the frequency in Hz.

$$V_L = i(2\pi fL)$$

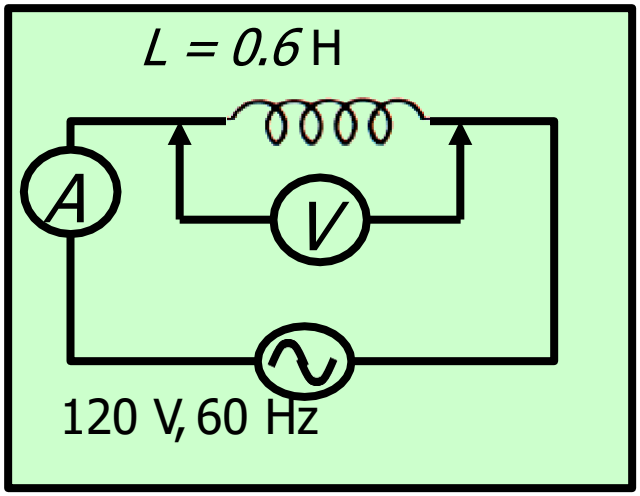
Ohm's law: $V_L = i_{eff} X_L$

Example 2: A coil having an inductance of 0.6 H is connected to a 120-V, 60 Hz ac source. Neglecting resistance, what is the effective current through the coil?

Reactance: $X_L = 2\pi fL$

$X_L = 2\pi(60\ \text{Hz})(0.6\ \text{H})$

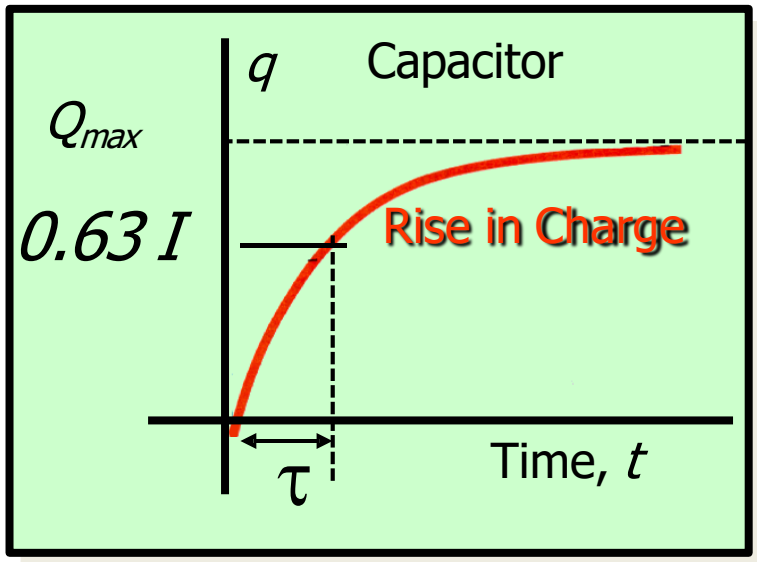
$X_L = 226\ \Omega$

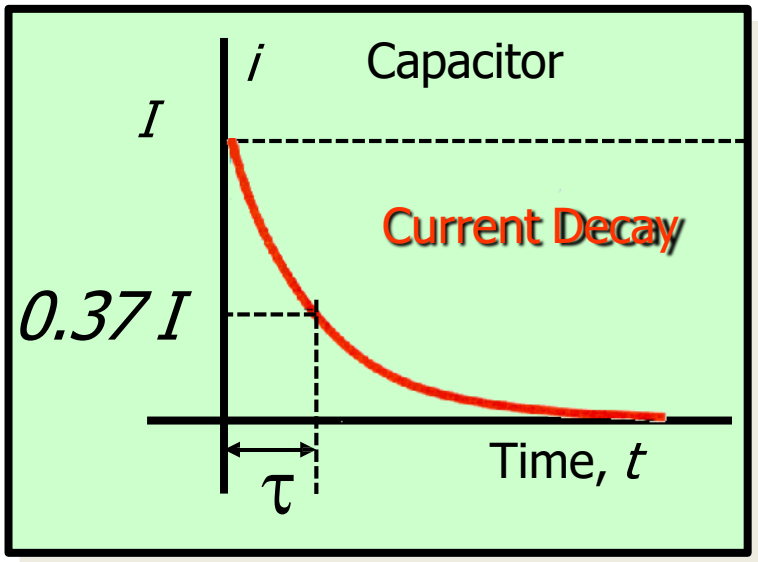


$i_{eff} = 0.531\ \text{A}$

Show that the peak current is $I_{max} = 0.750\ \text{A}$

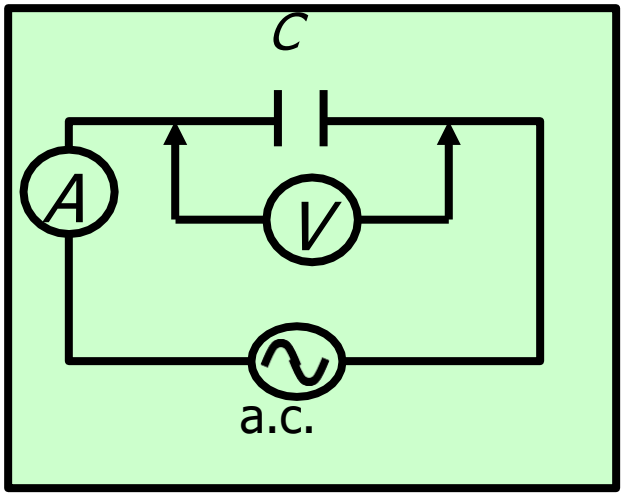
AC and Capacitance

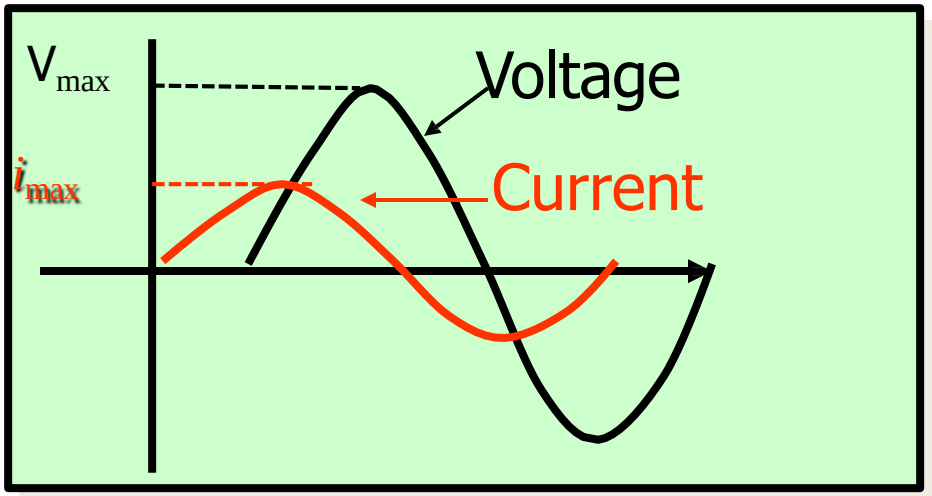




The voltage V peaks ¼ of a cycle after the current *i* reaches its maximum.
The voltage lags the current. Current *i* and V out of phase.

A Pure Capacitor in AC Circuit



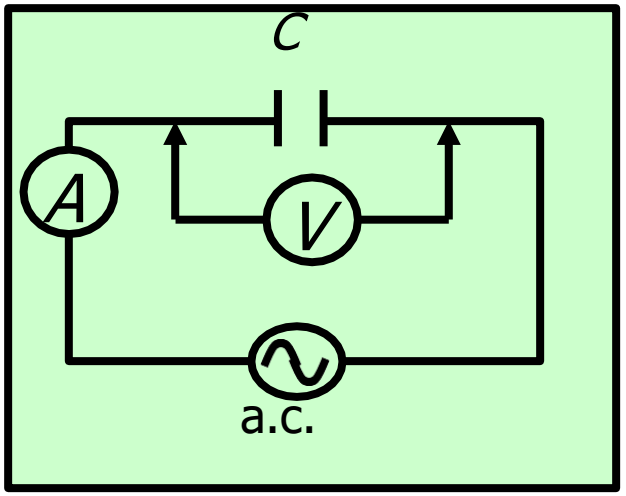


The voltage peaks 90^0 after the current peaks. One builds as the other falls and vice versa.

The diminishing current *i* builds charge on C which increases the back emf of V_C.

Capacitive Reactance

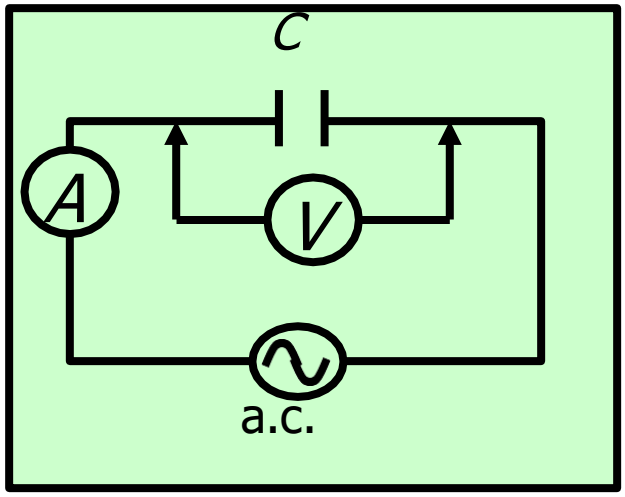
Energy gains and losses are also temporary for capacitors due to the constantly changing ac current.



No net power is lost in a complete cycle, even though the capacitor does provide nonresistive opposition (reactance) to the flow of ac current.

Capacitive reactance X_C is affected by both the capacitance and the frequency of the ac current.

Calculating Inductive Reactance



Capacitive Reactance:

$$X_C = \frac{1}{2\pi fC} \quad \text{Unit is the } \Omega$$

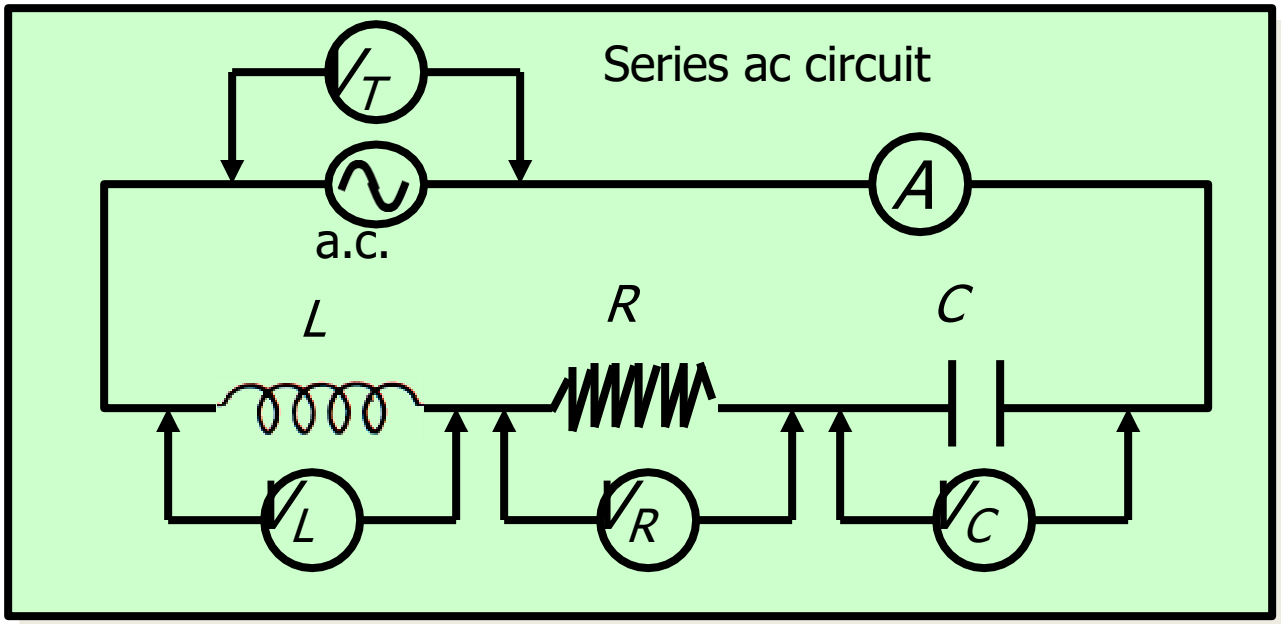
$$\text{Ohm's law: } V_C = iX_C$$

The voltage reading V in the above circuit at the instant the ac current is i can be found from the inductance in F and the frequency in Hz.

$$V_L = \frac{i}{2\pi fL}$$

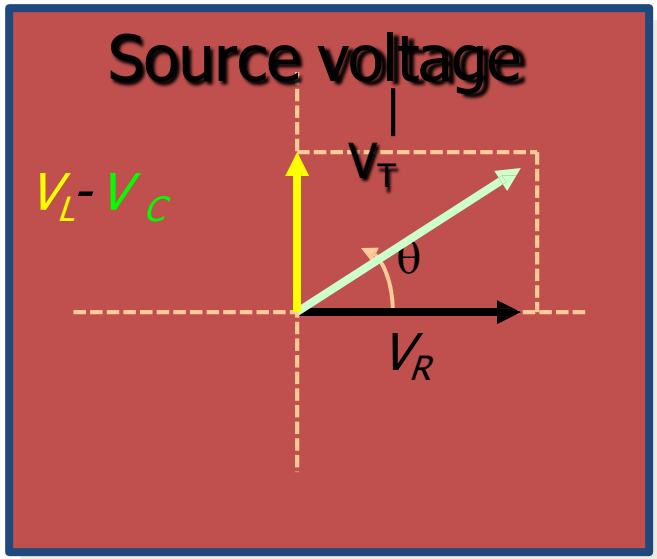
Ohm's law: $V_C = i_{eff} X_C$

Series LRC Circuits



Consider an inductor *L*, a capacitor *C*, and a resistor *R* all connected in series with an ac source. The instantaneous current and voltages can be measured with meters.

Calculating Total Source Voltage



Treating as vectors, we find:

$$V_T = \sqrt{V_R^2 + (V_L - V_C)^2}$$

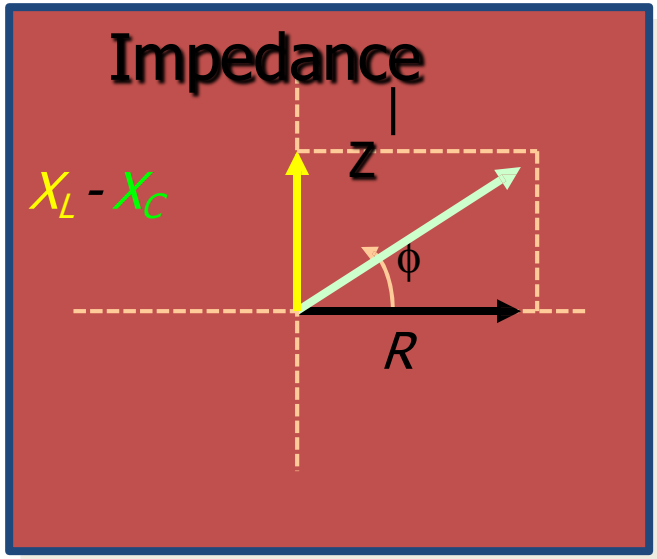
$$\tan \phi = \frac{V_L - V_C}{V_R}$$

Now recall that: $V_R = iR$; $V_L = iX_L$; and $V_C = iX_C$

Substitution into the above voltage equation gives:

$$V_T = i\sqrt{R^2 + (X_L - X_C)^2}$$

Impedance in an AC Circuit



$$V_T = i\sqrt{R^2 + (X_L - X_C)^2}$$

Impedance Z is defined:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

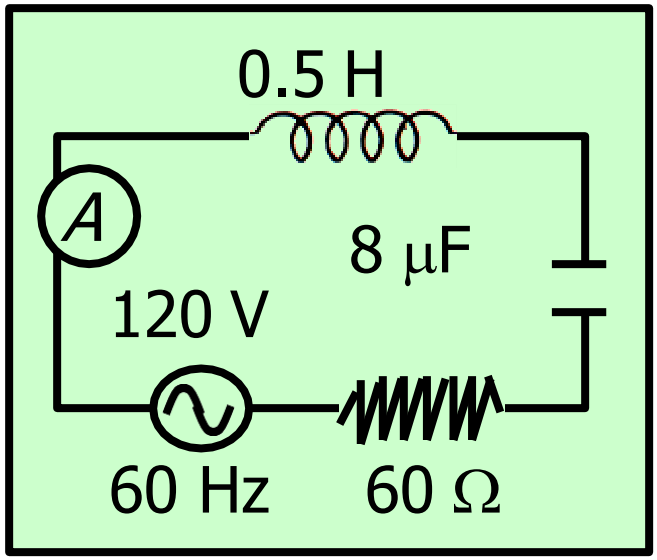
Ohm's law for ac current and impedance:

$$V_T = iZ \quad \text{or} \quad i = \frac{V_T}{Z}$$

The impedance is the combined opposition to ac current consisting of both resistance and reactance.

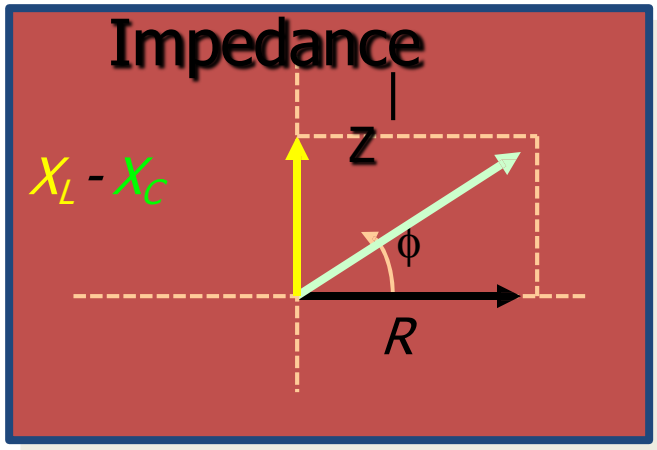
Example 4: Find the effective current and the phase angle for the previous example.

$X_L = 226\ \Omega$; $X_C = 332\ \Omega$; $R = 60\ \Omega$; $Z = 122\ \Omega$



$i_{eff} = 0.985$ A

Next we find the phase angle:



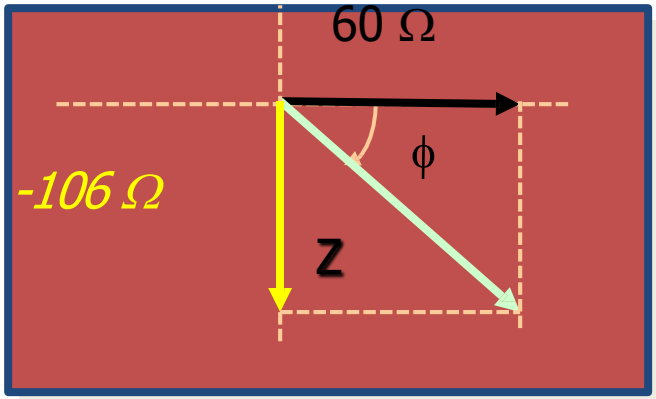
$X_L - X_C = 226 - 332 = -106\ \Omega$

$R = 60\ \Omega$

$$\tan\phi = \frac{X_L - X_C}{R}$$

Continued . . .

Example 4 (Cont.): Find the phase angle ϕ for the previous example.



$X_L - X_C = 226 - 332 = -106\ \Omega$

$R = 60\ \Omega$

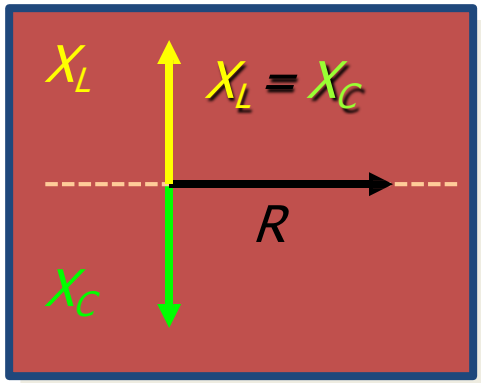
$$\tan\phi = \frac{X_L - X_C}{R}$$

$\phi = -60.5^0$

The negative phase angle means that the ac voltage lags the current by 60.5^0. This is known as a capacitive circuit.

Resonant Frequency

Because inductance causes the voltage to lead the current and capacitance causes it to lag the current, they tend to cancel each other out.



Resonance (Maximum Power) occurs when $X_L = X_C$

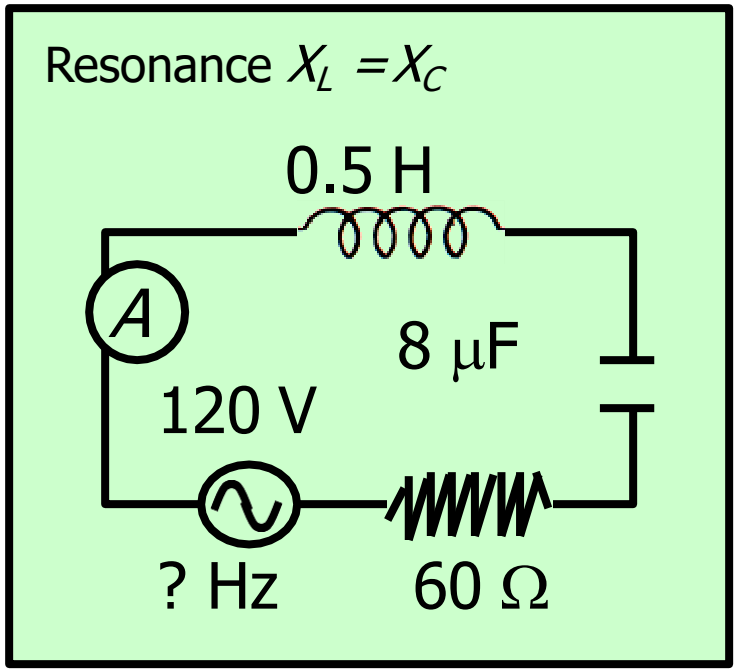
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$$

Resonant f_r $X_L = X_C$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Example 5: Find the resonant frequency for the previous circuit example: L = .5 H, C = 8 μF

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

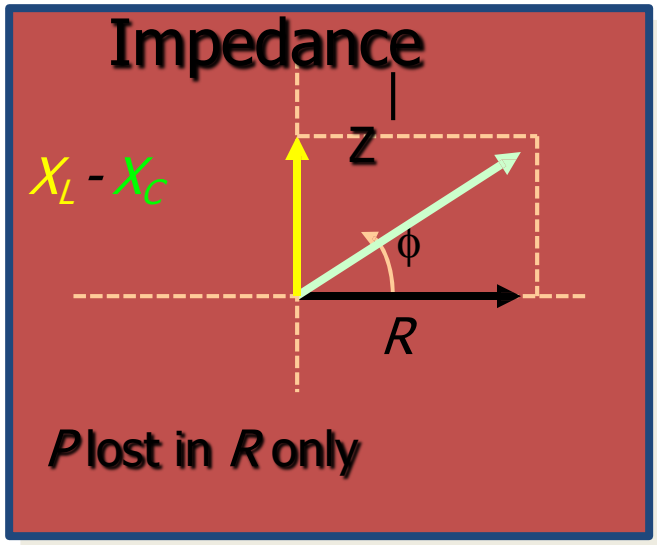


Resonant $f_r = 79.6$ Hz

At resonant frequency, there is zero reactance (only resistance) and the circuit has a phase angle of zero.

Power in an AC Circuit

No power is consumed by inductance or capacitance. Thus power is a function of the component of the impedance along resistance:



In terms of ac voltage:

$$P = iV \cos \phi$$

In terms of the resistance R:

$$P = i^2 R$$

The fraction Cos ϕ is known as the power factor.

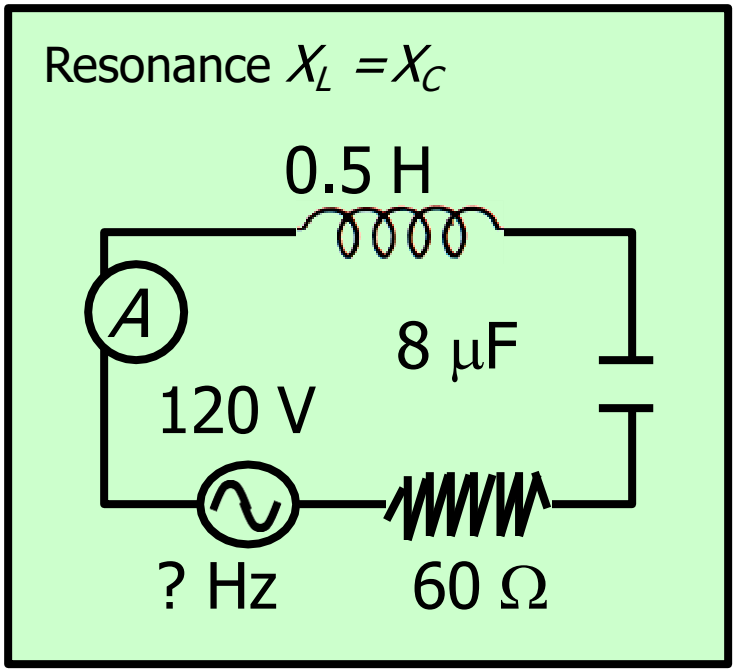
Example 6: What is the average power loss for the previous example: $V = 120$ V, $\phi = -60.5^0$, $i = 90.5$ A, and R $= 60\Omega$.

$P = i^2R = (0.0905\ A)^2(60\ \Omega)$

Average P = 0.491 W

The power factor is: Cos 60.5^0

Cos ϕ = 0.492 or 49.2%

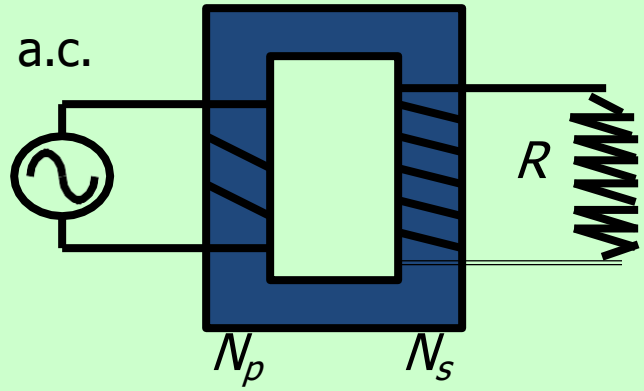


The higher the power factor, the more efficient is the circuit in its use of ac power.

The Transformer

A transformer is a device that uses induction and ac current to step voltages up or down.

An ac source of emf E_p is connected to primary coil with N_p turns. Secondary has N_s turns and emf of E_s.

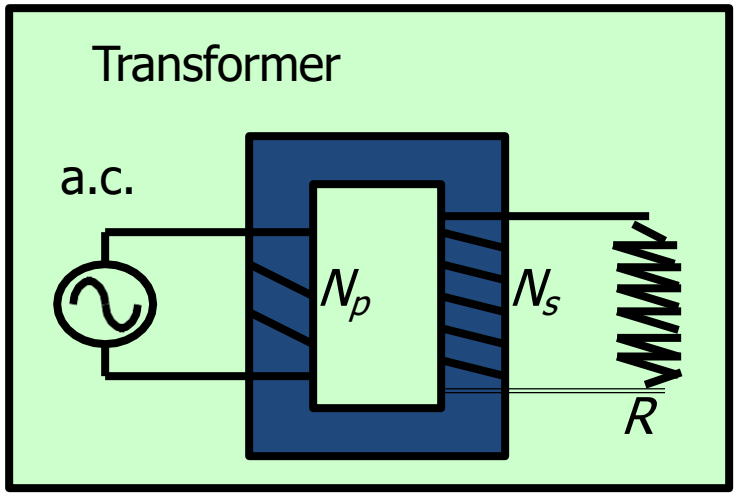


Induced emf's are:

$$\mathbf{E}_P = -N_P \frac{\Delta\Phi}{\Delta t}$$

$$\mathbf{E}_S = -N_S \frac{\Delta\Phi}{\Delta t}$$

Transformers (Continued):



$$\mathcal{E}_P = -N_P \frac{\Delta\Phi}{\Delta t}$$

$$\mathcal{E}_S = -N_S \frac{\Delta\Phi}{\Delta t}$$

Recognizing that $\Delta\phi/\Delta t$ is the same in each coil, we divide first relation by second and obtain:

The transformer equation:

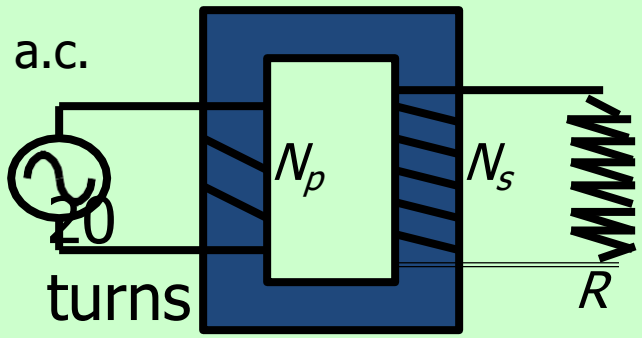
$$\frac{\mathcal{E}_P}{\mathcal{E}_S} = \frac{N_P}{N_S}$$

Example 7: A generator produces 10 A at 600 V. The primary coil in a transformer has 20 turns. How many secondary turns are needed to step up the voltage to 2400 V?

Applying the transformer equation:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$

I_p = 10 A; V_p = 600 V

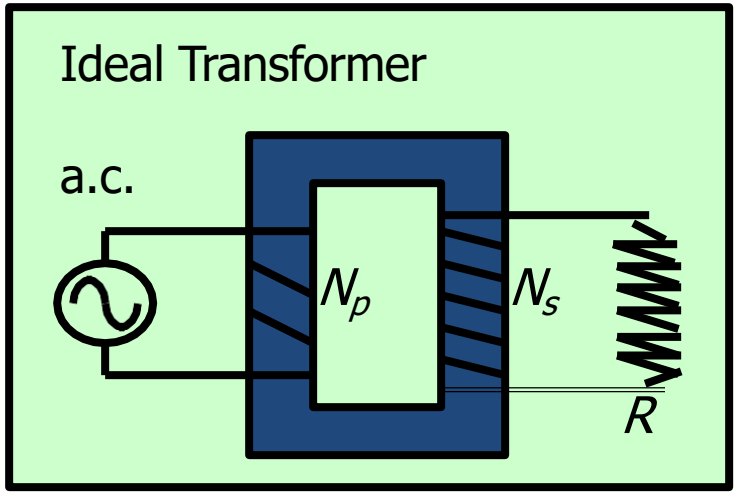


N_S = 80 turns

This is a step-up transformer; reversing coils will make it a step-down transformer.

Transformer Efficiency

There is no power gain in stepping up the voltage since voltage is increased by reducing current. In an ideal transformer with no internal losses:
here is no power gain in stepping up the voltage since voltage is increased by reducing current. In an ideal transformer with no internal losses:



An ideal transformer:

$$E_P i_P = E_S i_S \quad \text{or} \quad \frac{i_P}{i_s} = \frac{E_S}{E_P}$$

The above equation assumes no internal energy losses due to heat or flux changes. Actual efficiencies are usually between 90 and 100%.

Summary

Effective current: $i_{eff} = 0.707\ i_{max}$

Effective voltage: $V_{eff} = 0.707\ V_{max}$

Inductive Reactance:

$$X_L = 2\pi fL \quad \text{Unit is the } \Omega$$

$$\text{Ohm's law: } V_L = i X_L$$

Capacitive Reactance:

$$X_C = \frac{1}{2\pi fC} \quad \text{Unit is the } \Omega$$

$$\text{Ohm's law: } V_C = i X_C$$

Summary (Cont.)

$$V_T = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$V_T = iZ \quad \text{or} \quad i = \frac{V_T}{Z}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Summary (Cont.)

Power in AC Circuits:

In terms of ac voltage:

$$P = iV \cos \phi$$

In terms of the resistance R:

$$P = i^2 R$$

Transformers:

$$\frac{\mathcal{E}_P}{\mathcal{E}_S} = \frac{N_P}{N_S} \qquad \mathcal{E}_P i_P = \mathcal{E}_S i_S$$

UNIT- IV

RESONANCE AND MAGNETIC CIRCUIT

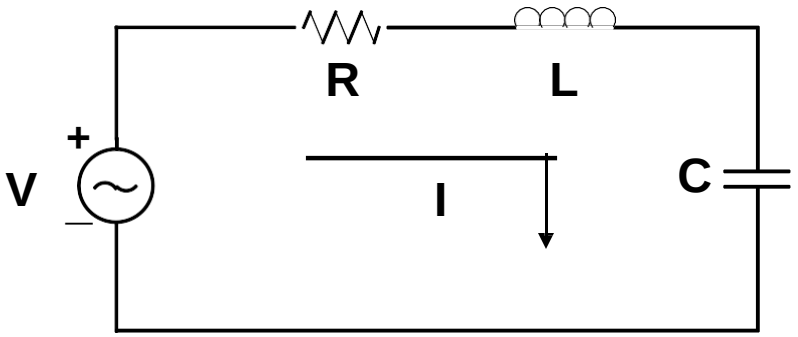
Resonance In Electric Circuits

- Any passive electric circuit will resonate if it has an inductor and capacitor.
- Resonance is characterized by the input voltage and current being in phase. The driving point impedance (or admittance) is completely real when this condition exists.
- In this presentation we will consider (a) series resonance, and (b) parallel resonance.

Series Resonance

Consider the series RLC circuit shown below.

$V = V_M \angle 0$



The input impedance is given by:

$$Z = R + j\left(wL - \frac{1}{wC}\right)$$

The magnitude of the circuit current is;

$$I = |\bar{I}| = \frac{V_m}{\sqrt{R^2 + \left(wL - \frac{1}{wC}\right)^2}}$$

Resonance occurs when.

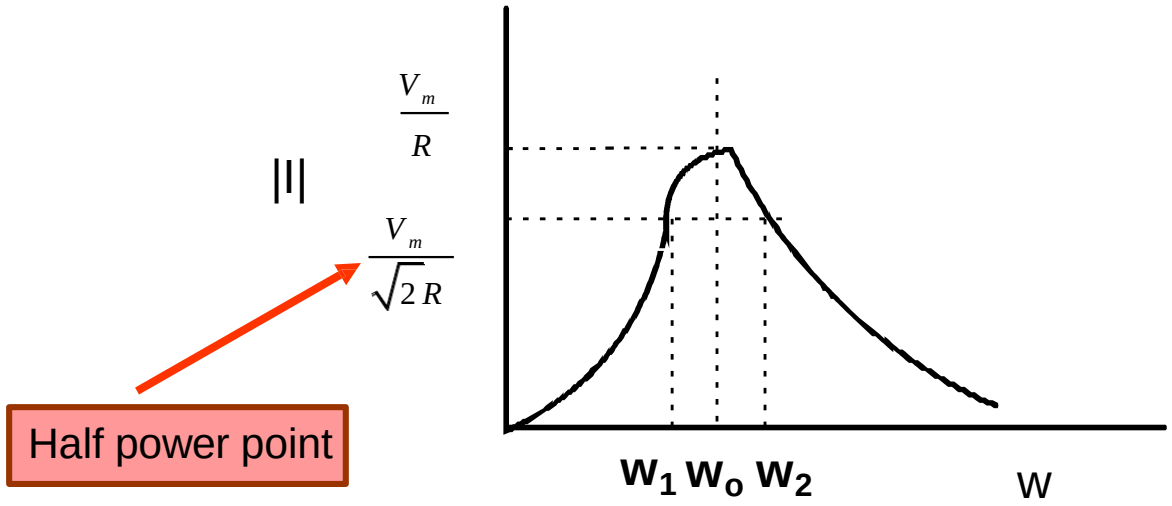
$$wL = \frac{1}{wC}$$

At resonance we designate w as w_o and write;

$$w_o = \frac{1}{\sqrt{LC}}$$

This is *an* ***important equation* to remember. It applies to both series And parallel resonant circuits.**

The magnitude of the current response for the series resonance circuit is as shown below.



Bandwidth:

$BW = w_{BW} = w_2 - w_1$

The peak power delivered to the circuit is;

$$P = \frac{V_m^{\,2}}{R}$$

The so-called half-power is given when $I = \frac{V_m}{\sqrt{2}R}$.

We find the frequencies, w_1 and w_2, at which this half-power occurs by using;

$$\sqrt{2}R = \sqrt{R^2 + \left(wL - \frac{1}{wC}\right)^2}$$

After some insightful algebra one will find two frequencies at which the previous equation is satisfied, they are:

$$w_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

and

$$w_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

The two half-power frequencies are related to the resonant frequency by

$$w_o = \sqrt{w_1 w_2}$$

The bandwidth of the series resonant circuit is given by;

$$BW = w_b = w_2 - w_1 = \frac{R}{L}$$

We define the Q (quality factor) of the circuit as;

$$Q = \frac{w_o L}{R} = \frac{1}{w_o R C} = \frac{1}{R}\sqrt{\left(\frac{L}{C}\right)}$$

Using Q, we can write the bandwidth as;

$$BW = \frac{w_o}{Q}$$

These are all important relationships.

An Observation:

If Q > 10, one can safely use the approximation;

$$w_1 = w_o - \frac{BW}{2} \qquad and \qquad w_2 = w_o + \frac{BW}{2}$$

These are useful approximations.

An Observation:

By using $Q = w_o L/R$ in the equations for w_1 and w_2 we have;

$$w_1 = w_o \left[\frac{-1}{2Q} + \sqrt{\left(\frac{1}{2Q} \right)^2 + 1} \right]$$

and

$$w_2 = w_o \left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q} \right)^2 + 1} \right]$$

In order to get some feel for how the numerical value of Q influences the resonant and also get a better appreciation of the s-plane, we consider the following example.

It is easy to show the following for the series RLC circuit.

$$\frac{I(s)}{V(s)} = \frac{1}{Z(s)} = \frac{\frac{1}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

In the following example, three cases for the about transfer function will be considered. We will keep w_o the same for all three cases.
The numerator gain,k, will (a) first be set k to 2 for the three cases, then (b) the value of k will be set so that each response is 1 at resonance.

An Example Illustrating Resonance:

The 3 transfer functions considered are:

Case 1:

$$\frac{ks}{s^2 + 2s + 400}$$

Case 2:

$$\frac{ks}{s^2 + 5s + 400}$$

Case 3:

$$\frac{ks}{s^2 + 10s + 400}$$

An Example Illustrating Resonance:

The poles for the three cases are given below.

Case 1:

$$s^2 + 2s + 400 = (s + 1 + j19.97)(s + 1 - j19.97)$$

Case 2:

$$s^2 + 5s + 400 = (s + 2.5 + j19.84)(s + 2.5 - j19.84)$$

Case 3:

$$s^2 + 10s + 400 = (s + 5 + j19.36)(s + 5 - j19.36)$$

Comments:

Observe the denominator of the CE equation.

$$s^2 + \frac{R}{L}s + \frac{1}{LC}$$

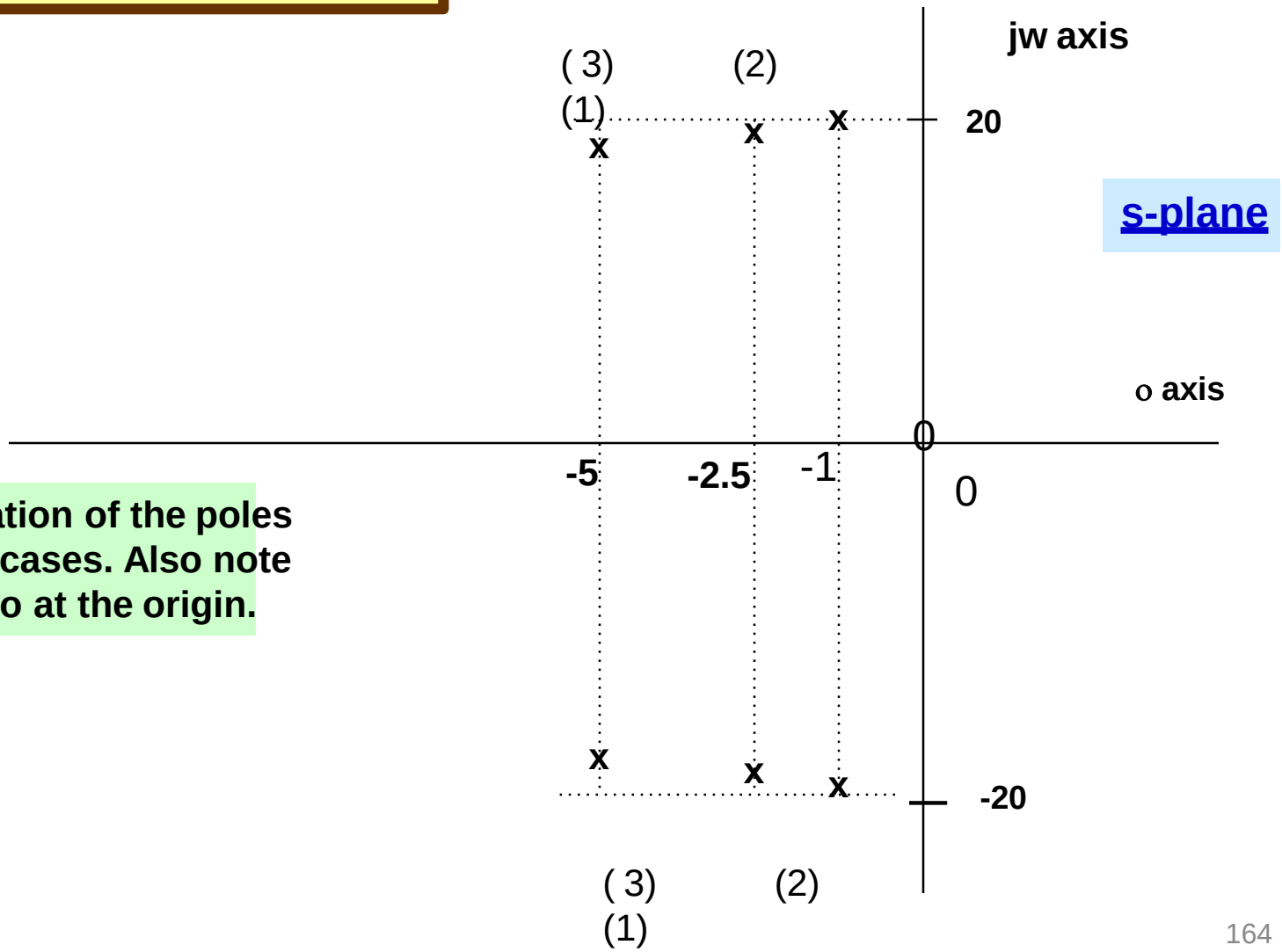
Compare to actual characteristic equation for *Case 1*:

$$s^2 + 2s + 400$$

$$w_o^2 = 400 \longrightarrow w = 20 \quad \text{rad/sec}$$

$$BW = \frac{R}{L} = 2 \quad \text{rad/sec} \longrightarrow Q = \frac{w_o}{BW} = 10$$

Poles and Zeros In the s-plane:



[s-plane]

Note the location of the poles for the three cases. Also note there is a zero at the origin.

Comments:

The frequency response starts at the origin in the s-plane. At the origin the transfer function is zero because there is a zero at the origin.

As you get closer and closer to the complex pole, which has a j parts in the neighborhood of 20, the response starts to increase.

The response continues to increase until we reach w = 20. From there on the response decreases.

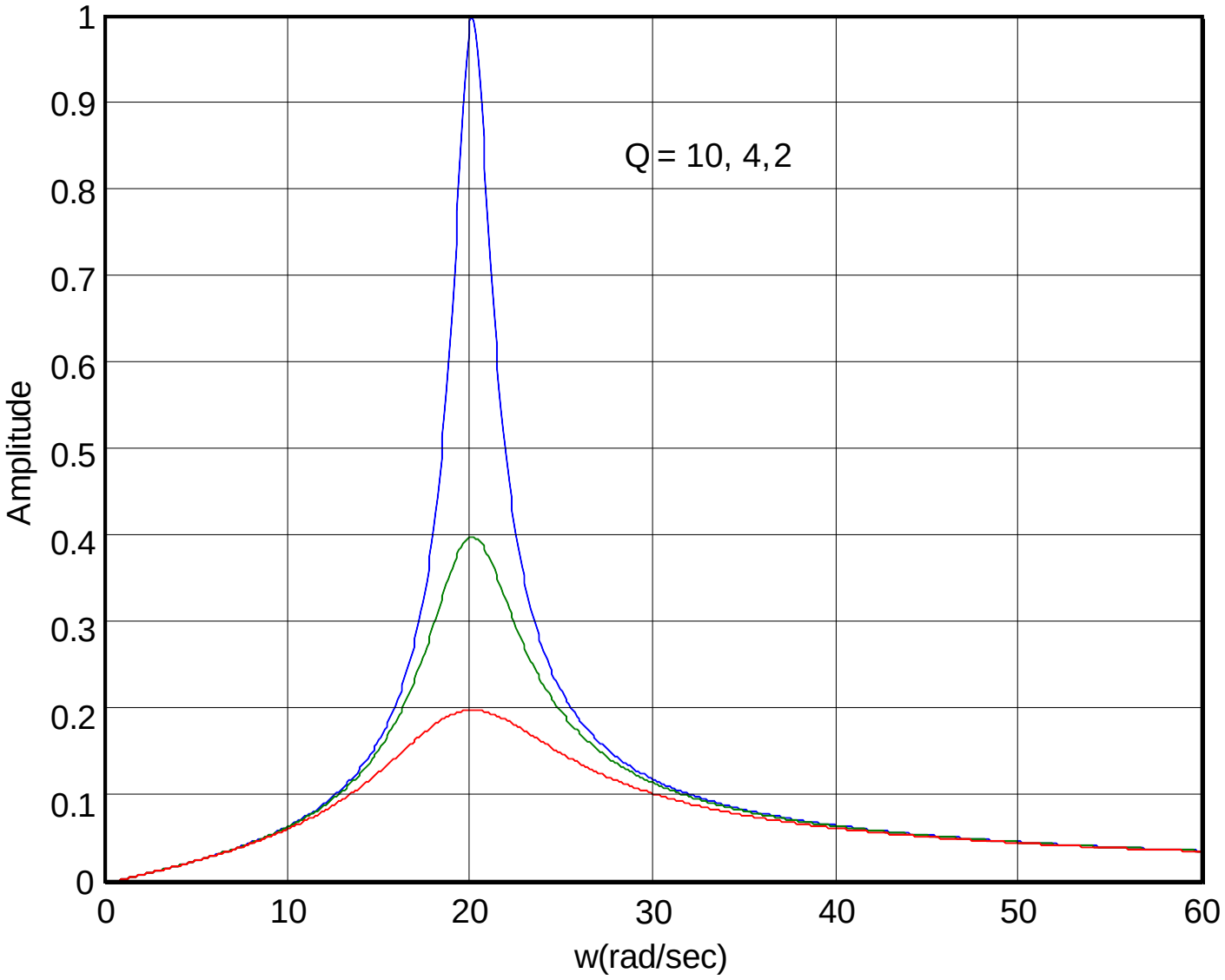
We should be able to reason through why the response has the above characteristics, using a graphical approach.

Matlab Program For The Study:

```
% name of program is freqtest.m
% written for 202 S2002, wlg
%CASE ONE DATA:
K = 2;
num1 = [K 0];
den1 = [1 2  400];

num2 = [K 0];
den2 = [1 5 400];

num3 = [K 0];
den3 = [1 10 400];

w = .1:.1:60;
```

```
grid
H1 = bode(num1,den1,w);
magH1=abs(H1);

H2 = bode(num2,den2,w);
magH2=abs(H2);

H3 = bode(num3,den3,w);
magH3=abs(H3);

plot(w,magH1, w, magH2, w,magH3)
grid
xlabel('w(rad/sec)')
ylabel('Amplitude')
gtext('Q = 10, 4, 2')
```

Program Output



Comments: cont.

From earlier work:

$$w_1, w_2 = w_o \left[\frac{\pm 1}{2Q} + \sqrt{\left(\frac{1}{2Q} \right)^2 + 1} \right]$$

With Q = 10, this gives;

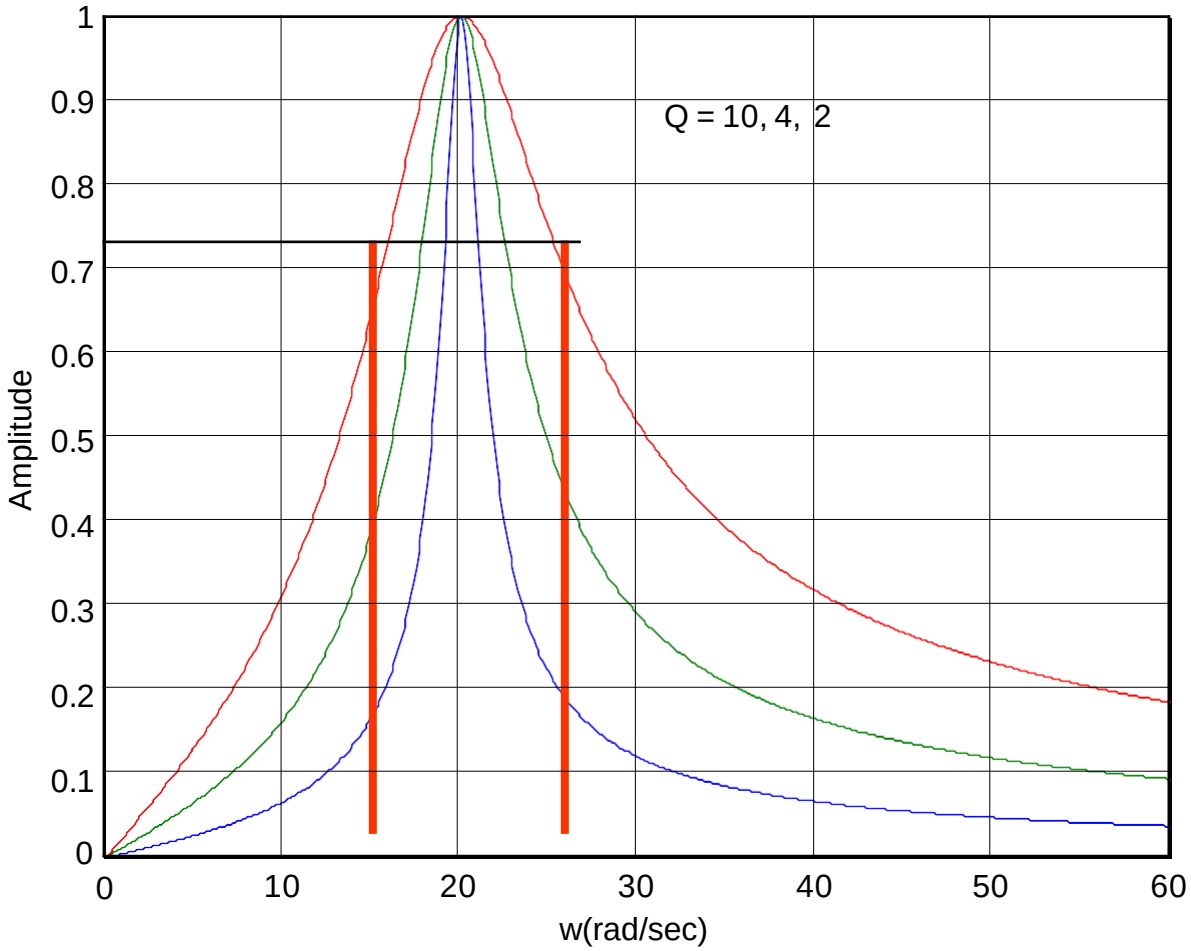
w_1= 19.51 rad/sec, w_2 = 20.51 rad/sec

Compare this to the approximation:

$w_1 = w_0 - BW = 20 - 1 = 19$ rad/sec, $w_2 = 21$ rad/sec

So basically we can find all the series resonant parameters if we are given the numerical form of the CE of the transfer function.

Next Case: Normalize all responses to 1 at w_o



Three dB Calculations:

Now we use the analytical expressions to calculate w_1 and w_2. We will then compare these values to what we find from the Matlab simulation.

Using the following equations with Q = 2,

$$w_1, w_2 = w_o \left[\frac{-1}{2Q} \pm w_o \sqrt{\left(\frac{1}{2Q} \right)^2 + 1} \right]$$

we find,

w_1 = 15.62 rad/sec

w_2 = 21.62 rad/sec

Checking w_1 and w_2

(cut-outs from the simulation)

w_1 →

15.3000	0.6779
15.4000	0.6871
15.5000	0.6964
15.6000	0.7057
15.7000	0.7150
15.8000	0.7244

w_2 →

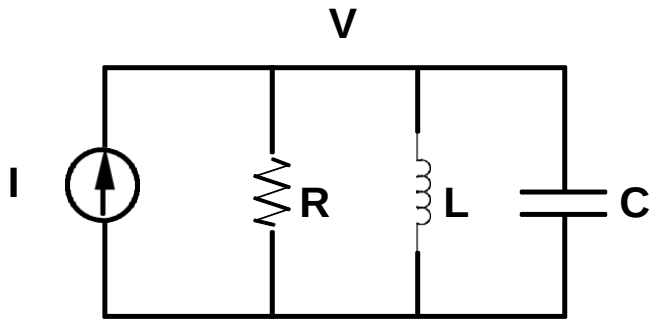
25.3000	0.7254
25.4000	0.7195
25.5000	0.7137
25.6000	0.7080
25.7000	0.7023
25.8000	0.6967
25.9000	0.6912

This verifies the previous calculations.

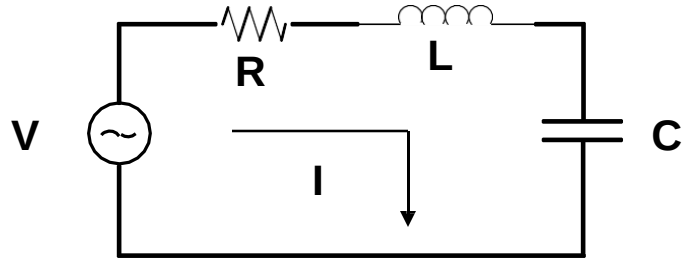
Now we shall look at Parallel Resonance.

Background

Consider the circuits shown below:



$$I = V\left[\frac{1}{R} + jwC + \frac{1}{jwL}\right]$$



$$V = I\left[R + jwL + \frac{1}{jwC}\right]$$

Duality

$$I = V\left[\frac{1}{R} + jwC + \frac{1}{jwL}\right] \qquad V = I\left[R + jwL + \frac{1}{jwC}\right]$$

We notice the above equations are the same provided:

$$I \longleftrightarrow V$$

$$R \longleftrightarrow \frac{1}{R}$$

$$L \longleftrightarrow C$$

If we make the inner-change, then one equation becomes the same as the other.

For such case, we say the one circuit is the dual of the other.

Parallel Resonance

Background

What this means is that for all the equations we have derived for the parallel resonant circuit, we can use for the series resonant circuit provided we make the substitutions:

$$R \quad replaced \quad be \quad \frac{1}{R}$$

$$L \quad replaced \quad by \quad C$$

$$C \quad replaced \quad by \quad L$$

Parallel Resonance

$$w_o = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{w_o L}{R}$$

$$BW = (w_2 - w_1) = w_{BW} = \frac{R}{L}$$

$$w_1, w_2 = \left[\frac{\mp R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right]$$

$$w_1, w_2 = w_o \left[\frac{\mp 1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$$

Series Resonance

$$w_o = \frac{1}{\sqrt{LC}}$$

$$Q = w_o RC$$

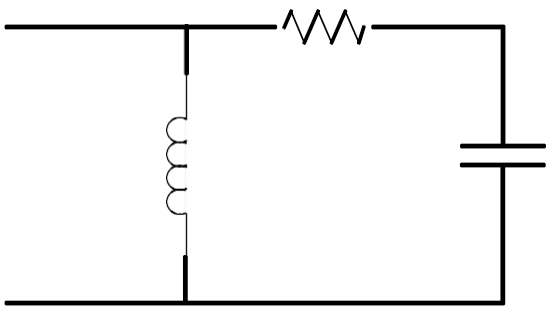
$w_1 w_2$

$$BW = w_{BW} = \frac{1}{RC}$$

$$w_1, w_2 = \left[\frac{\mp 1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \right]$$

$$w_1, w_2 = w_o \left[\frac{\mp 1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$$

Example 1: **Determine the resonant frequency for the circuit below.**



$$Z_{IN} = \frac{jwL\ (R + \frac{1}{jwC})}{R + jwL + \frac{1}{jwC}} = \frac{(-w^2 LRC + jwL)}{(1 - w^2 LC) + jwRC}$$

At resonance, the phase angle of Z must be equal to zero.

Analysis

$$\frac{(-w^2 LRC + jwL)}{(1 - w^2 LC) + jwRC}$$

For zero phase;

$$\frac{wL}{(-w^2 LCR)} = \frac{wRC}{(1 - w^2 LC}$$

This gives;

$$w^2 LC - w^2 R^2 C^2 = 1$$

or

$$w_o = \frac{1}{\sqrt{(LC - R^2 C^2)}}$$

Example 2:

A series RLC resonant circuit has a resonant frequency admittance of 2×10^{-2} S(mohs). The Q of the circuit is 50, and the resonant frequency is 10,000 rad/sec. Calculate the values of R, L, and C. Find the half-power frequencies and the bandwidth.

First, R = 1/G = 1/(0.02) = 50 ohms.

Second, from $Q = \frac{w_o L}{R}$, we solve for L, knowing Q, R, and w_o to find L = 0.25 H.

Third, we can use $C = \frac{Q}{w_o R} = \frac{50}{10,000 \times 50} = 100 \quad \mu F$

Example 2: (continued)

Fourth: We can use $w_{BW} = \frac{w_o}{Q} = \frac{1x10^4}{50} = 200 \ rad/\sec$

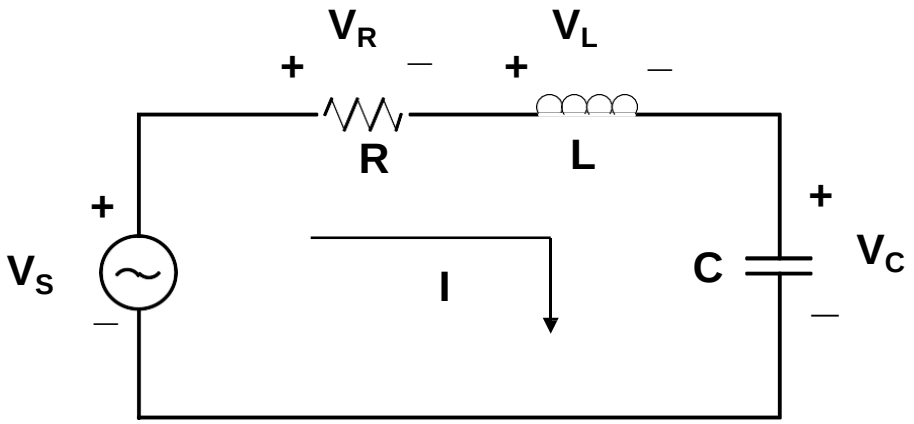
and

Fifth: Use the approximations;

$w_1 = w_o - 0.5w_{BW} = 10{,}000 - 100 = 9{,}900$ rad/sec

$w_2 = w_o - 0.5w_{BW} = 10{,}000 + 100 = 10{,}100$ rad/sec

Peak Voltages and Resonance:



We know the following:

- When $w = w_o = \frac{1}{\sqrt{LC}}$, V_S and I are in phase, the driving point impedance is purely real and equal to R.

- A plot of |I| shows that it is maximum at $w = w_o$. We know the standard equations for series resonance applies: Q, w_{BW}, etc.

Reflection:

- A question that arises is what is the nature of V_R, V_L, and V_C? A little reflection shows that V_R is a peak value at w_o. But we are not sure about the other two voltages. We know that at resonance they are equal and they have a magnitude of QxV_S.

- Irwin shows that the frequency at which the voltage across the capacitor is a maximum is given by;

$$w_{max} = w_o \sqrt{1 - \frac{1}{2Q^2}}$$

- The above being true, we might ask, what is the frequency at which the voltage across the inductor is a maximum?

We answer this question by simulation

Series RLC Transfer Functions:

The following transfer functions apply to the series RLC circuit.

$$\frac{V_C(s)}{V_S(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\frac{V_L(s)}{V_S(s)} = \frac{s^2}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\frac{V_R(s)}{V_S(s)} = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Parameter Selection:

We select values of R, L. and C for this first case so that Q = 2 and w_o = 2000 rad/sec. Appropriate values are; R = 50 ohms, L = .05 H, C = 5μF. The transfer functions become as follows:

$$\frac{V_C}{V_S} = \frac{4x10^6}{s^2 + 1000s + 4x10^6}$$

$$\frac{V_L}{V_S} = \frac{s^2}{s^2 + 1000s + 4x10^6}$$

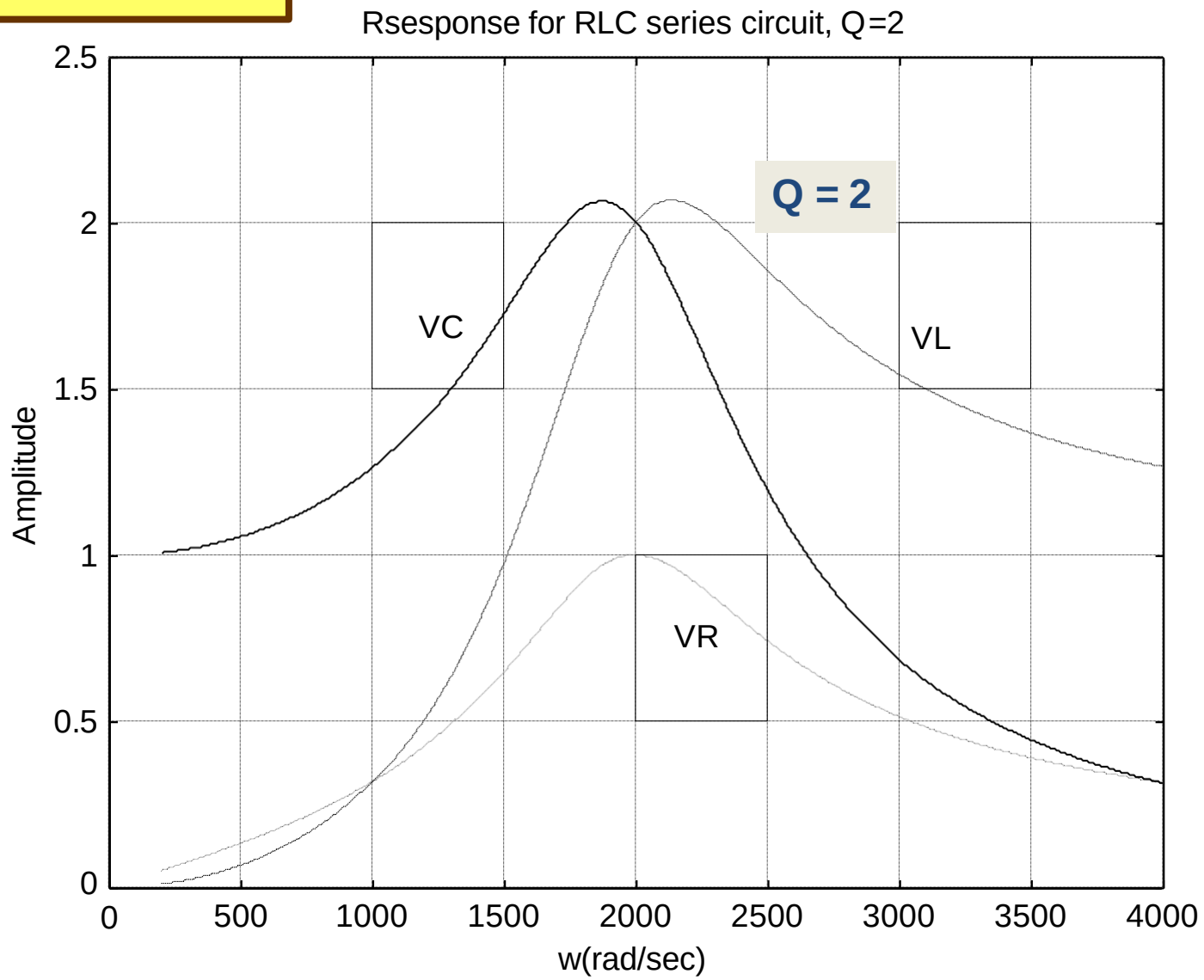
$$\frac{V_R}{V_S} = \frac{1000s}{s^2 + 1000s + 4x10^6}$$

Matlab Simulation:

```
% program is freqcompare.m
% written for 202 S2002, wlg

numC = 4e+6;
denC = [1 1000 4e+6];

numL = [1 0 0];
denL = [1 1000 4e+6];

numR = [1000 0];
denR = [1 1000 4e+6];

w = 200:1:4000;

grid
HC = bode(numC,denC,w);
magHC = abs(HC);
```

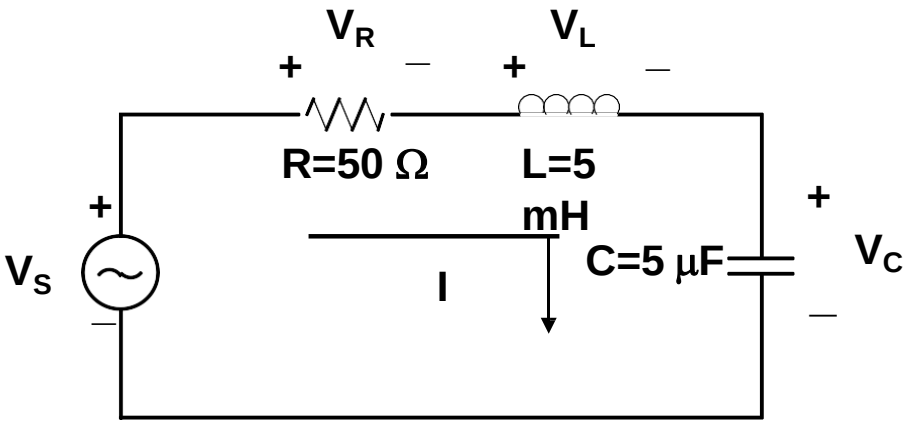
```
grid
HC = bode(numC,denC,w);
magHC = abs(HC);

HL = bode(numL,denL,w);
magHL = abs(HL);

HR = bode(numR,denR,w);
magHR = abs(HR);

plot(w,magHC,'k-', w, magHL,'k--', w, magHR, 'k:')
grid

xlabel('w(rad/sec)')
ylabel('Amplitude')
title(' Rsesponse for RLC series circuit, Q =2')

gtext('VC')
gtext('VL')
gtext(' VR')
```

Simulation Results



Analysis of the problem:

Given the previous circuit. Find Q, w_0, w_{max}, $|V_c|$ at w_o, and $|V_c|$ at w_{max}



Solution:

$$w_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50\ x10^{-2} x\ 5\ x10^{-6}}} = 2000 \quad rad \quad / \sec$$

$$Q = \frac{w_o L}{R} = \frac{2\ x10^{3} x\ 5x10^{-2}}{50} = 2$$

Problem Solution:

$$w_{MAX} = w_O \sqrt{1 - \frac{1}{2Q^2}} = 0.9354 \; w_o$$

$$|V_R| \; at \; w_O = Q \; |V_S| = 2x1 = 2 \; volts \; (peak)$$

$$|V_C| \; at \; w_{MAX} = \frac{Qx \; |V_S|}{\sqrt{1 - \frac{1}{4Q^2}}} = \frac{2}{0.968} = 2.066 \; volts \; (peak))$$

Now check the computer printout.

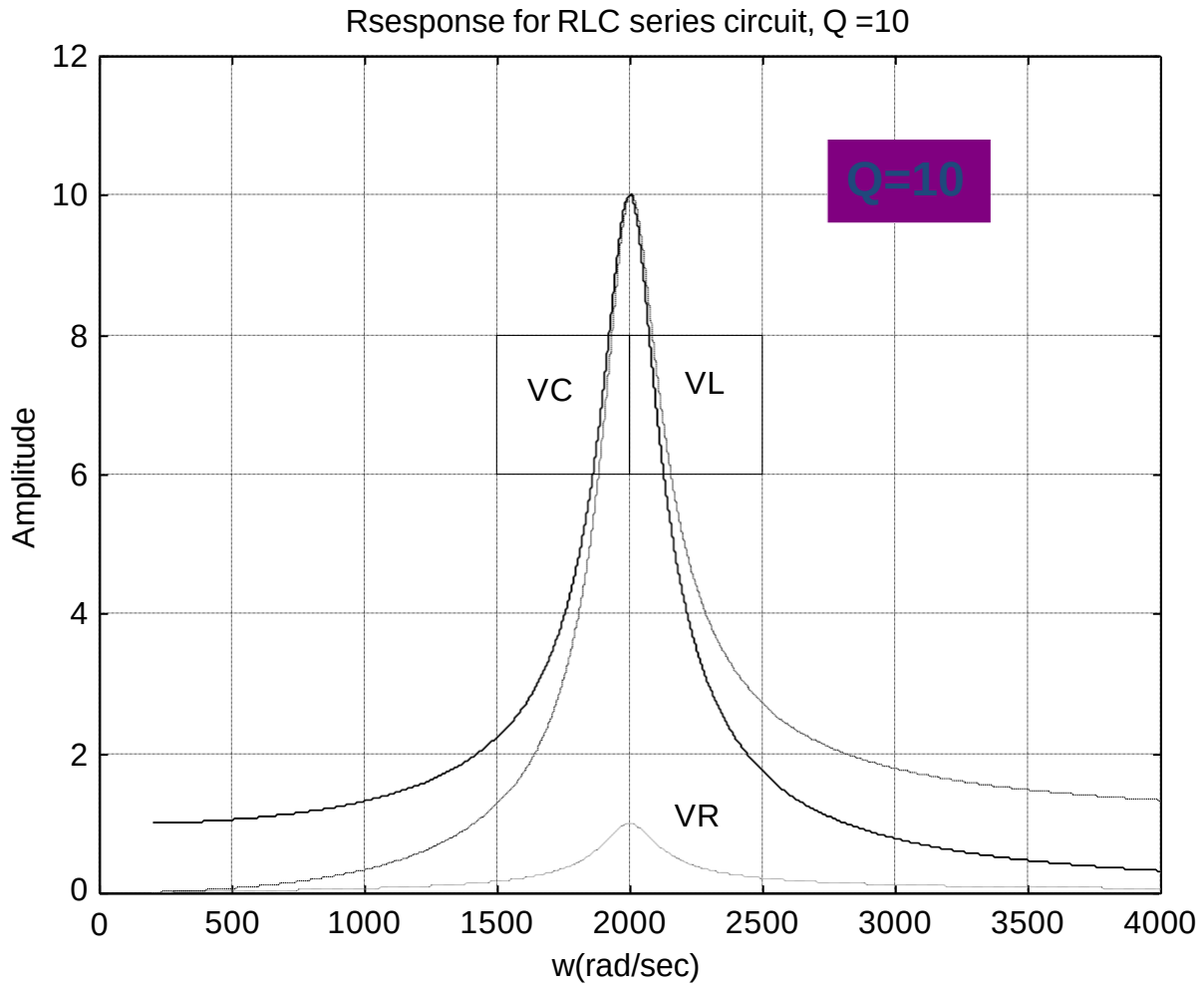
Problem Solution (Simulation):

1.0e+003 *

1.8600000	0.002065141
1.8620000	0.002065292
1.8640000	0.002065411
1.8660000	0.002065501
1.8680000	0.002065560
1.8700000	0.002065588
1.8720000	0.002065585
1.8740000	0.002065552
1.8760000	0.002065487
1.8780000	0.002065392
1.8800000	0.002065265
1.8820000	0.002065107
1.8840000	0.002064917

Maximum

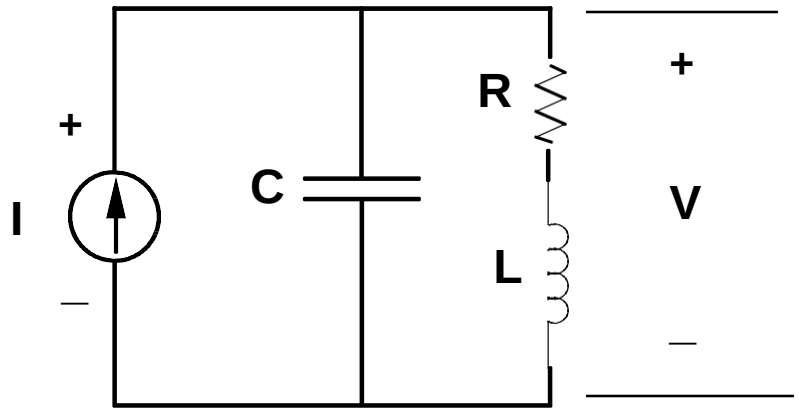
Simulation Results:



Observations From The Study:

- The voltage across the capacitor and inductor for a series RLC circuit is not at peak values at resonance for small Q ($Q < 3$).
- Even for $Q < 3$, the voltages across the capacitor and inductor are equal at resonance and their values will be QxV_S.
- For $Q > 10$, the voltages across the capacitors are for all practical purposes at their peak values and will be QxV_S.
- Regardless of the value of Q, the voltage across the resistor reaches its peak value at $w = w_o$.
- For high Q, the equations discussed for series RLC resonance can be applied to any voltage in the RLC circuit. For $Q < 3$, this is not true.

Given the following circuit:



- **We want to find the frequency, w_r, at which the transfer function for V/I will resonate.**

- **The transfer function will exhibit resonance when the phase angle between V and I are zero.**

The desired transfer functions is;

$$\frac{V}{I} = \frac{(1/sC)(R + sL)}{R + sL + 1/sC}$$

This equation can be simplified to;

$$\frac{V}{I} = \frac{R + sL}{LCs^2 + RCs + 1}$$

With s ⟶ jw

$$\frac{V}{I} = \frac{R + jwL}{(1 - w^2 LC) + jwR}$$

Resonant Condition:

For the previous transfer function to be at a resonant point, the phase angle of the numerator must be equal to the phase angle of the denominator.

$$\angle \theta_{num} = \angle \theta_{dem}$$

or,

$$\theta_{num} = \tan^{-1}\left(\frac{wL}{R}\right), \qquad \theta_{den} = \tan^{-1}\left(\frac{wRC}{(1 - w^2 LC)}\right).$$

Therefore;

$$\frac{wL}{R} = \frac{wRC}{(1 - w^2 LC)}$$

Resonant Condition Analysis:

Canceling the w's in the numerator and cross multiplying gives,

$$L\,(1 - w^2 L C) = R^2 C \qquad or \qquad w^2 L^2 C = L - R^2 C$$

This gives,

$$w_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

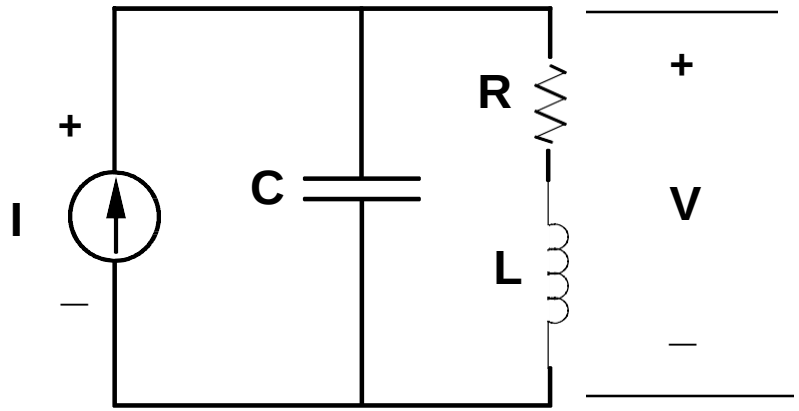
Notice that if the ratio of R/L is small compared to 1/LC, we have

$$w_r = w_o = \frac{1}{\sqrt{LC}}$$

Resonant Condition Analysis:

What is the significance of w_r and w_o in the previous two equations? Clearly w_r is a lower frequency of the two. To answer this question, consider the following example.

Given the following circuit with the indicated parameters. Write a Matlab program that will determine the frequency response of the transfer function of the voltage to the current as indicated.



Resonant Condition Analysis: Matlab Simulation:

We consider two cases:

Case 1:

R = 3 ohms
C = 6.25×10^{-5} F
L = 0.01 H

w_r = 2646 rad/sec

Case 2:

R = 1 ohms
C = 6.25×10^{-5} F
L = 0.01 H

w_r = 3873 rad/sec

For both cases,

w_o = 4000 rad/sec

Resonant Condition Analysis: Matlab Simulation:

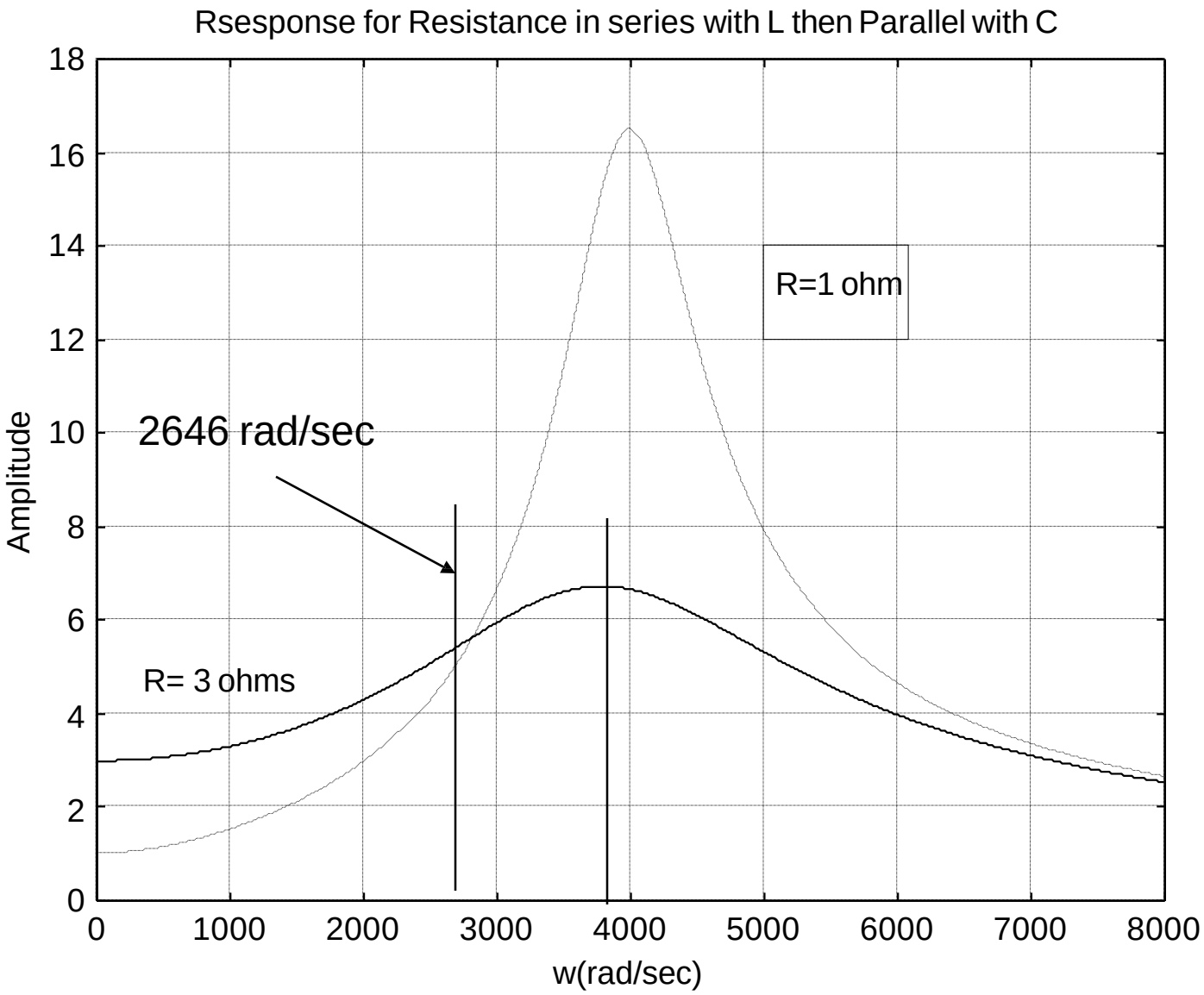
The transfer functions to be simulated are given below.

Case 1:

$$\frac{V}{I} = \frac{0.001s + 3}{6.25x10^{-8}s^2 + 1.875x10^{-7} + 1}$$

Case 2:

$$\frac{V}{I} = \frac{0.001s + 1}{6.25x10^{-8}s^2 + 6.25x10^{-5} + 1}$$


Rsesponse for Resistance in series with L then Parallel with C
Amplitude
w(rad/sec)
R=1 ohm
2646 rad/sec
R= 3 ohms
0
2
4
6
8
10
12
14
16
18
0
1000
2000
3000
4000
5000
6000
7000
8000

What can be learned from this example?

- **w_r does not seem to have much meaning in this problem. What is w_r if R = 3.99 ohms?**

- **Just because a circuit is operated at the resonant frequency does not mean it will have a peak in the response at the frequency.**

- **For circuits that are fairly complicated and can resonant, It is probably easier to use a simulation program similar to Matlab to find out what is going on in the circuit.**

MAGNETIC CIRCUITS

Magnetic and Electromagnetic Fields

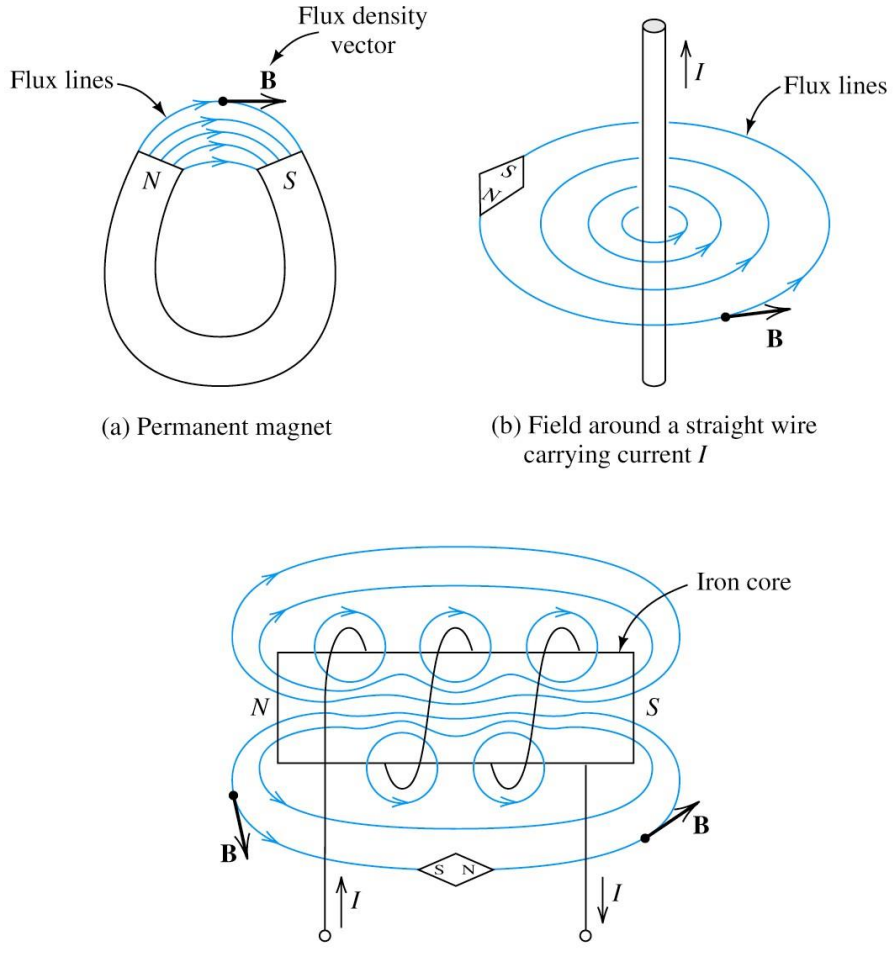
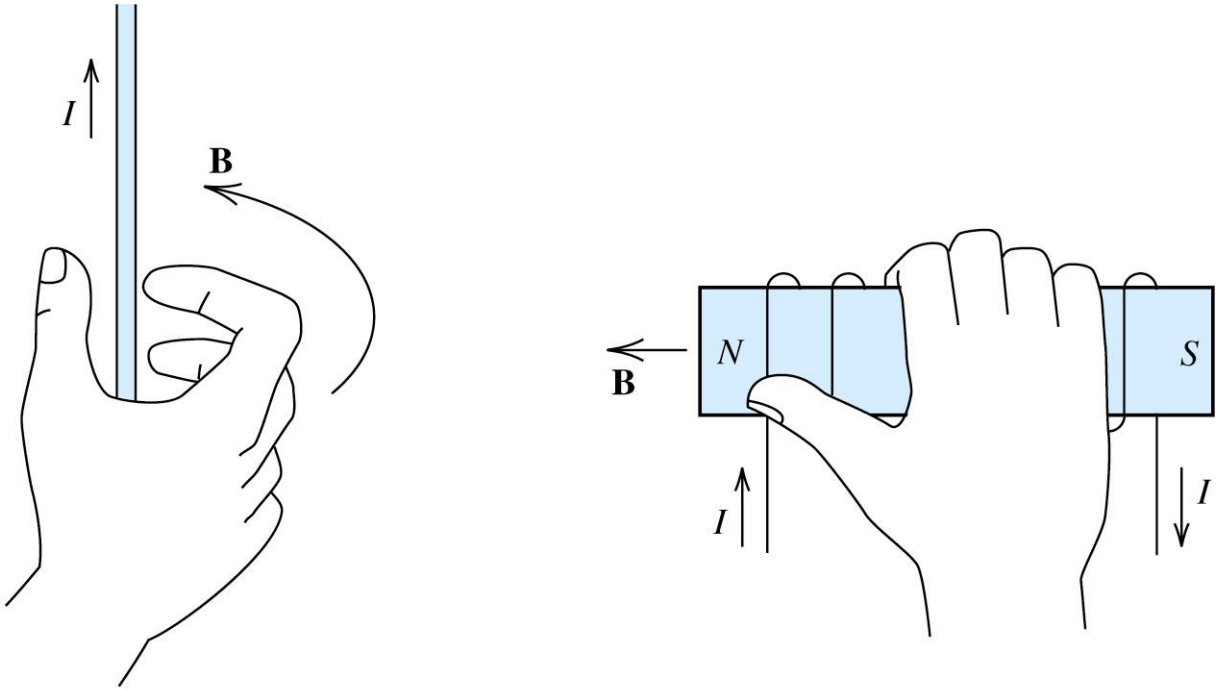


Figure 15.1 Magnetic fields can be visualized as lines of flux that form closed paths. Using a compass, we can determine the direction of the flux lines at any point. Note that the flux density vector **B** is tangent to the lines of flux.

Magnetic Materials

Iron, Cobalt and Nickel and various other alloys and compounds made using these three basic elements

Electric Current and Magnetic Field



(a) If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field

(b) If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

Figure 15.2 Illustrations of the right-hand rule.

A Few Definitions Related to Electromagnetic Field

Φ (Unit is Weber (Wb)) = Magnetic Flux Crossing a Surface of Area $\underline{A}$ in m^2.

B (Unit is Tesla (T)) = Magnetic Flux Density = Φ/A

H (Unit is Amp/m) = Magnetic Field Intensity = $\frac{B}{\mu}$

μ = permeability = $\mu_o \mu_r$

$\mu_o = 4\pi * 10^{-7}$ H/m (H $\Rightarrow$ Henry) = Permeability of free space (air)

μ_r = Relative Permeability

$\mu_r >> 1$ for Magnetic Material

Ampére‘s Law

The line integral of the magnetic field intensity around a closed path is equal to the sum of the currents flowing through the area enclosed by the path.

$$\oint \vec{H} \cdot \vec{dl} = \sum i$$

$$\vec{H} \cdot \vec{dl} = \left|\vec{H}\right| \left|\vec{dl}\right| \cos\ \theta$$

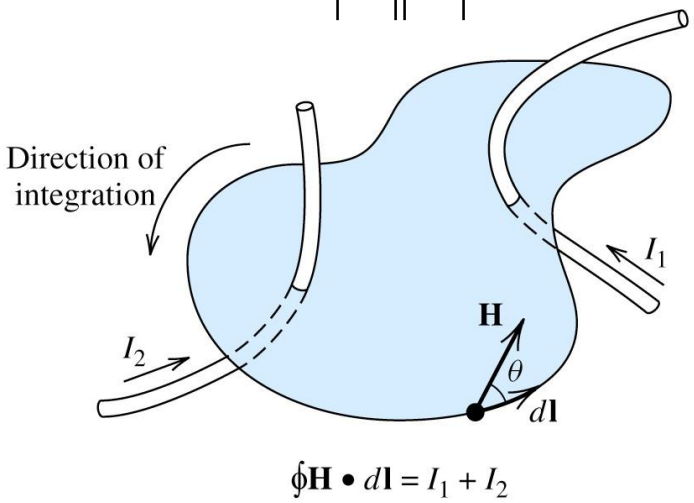


Figure 15.6 Ampère’s law states that the line integral of magnetic field intensity around a closed path is equal to the sum of the currents flowing through the surface bounded by the path.

Example of Ampére's Law

Find the magnetic field along a circular path around an infinitely long Conductor carrying I ampere of current.

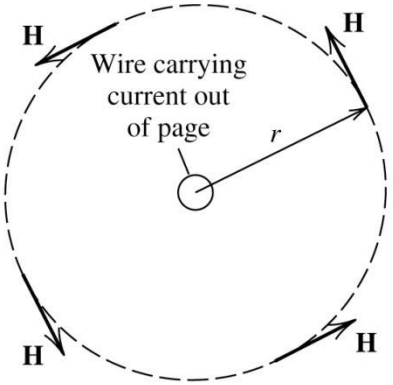
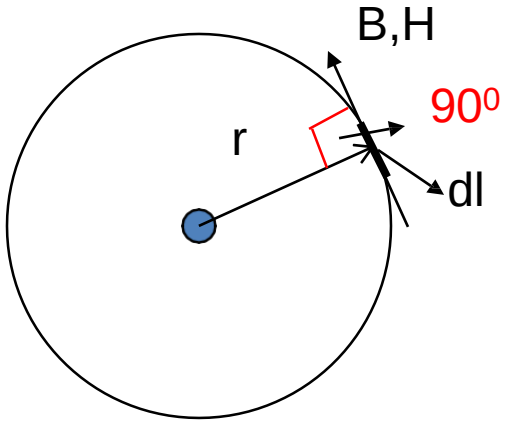


Figure 15.7 The magnetic field around a long straight wire carrying a current can be determined with Ampère's law aided by considerations of symmetry.



Since both $\vec{dl}$ and $\vec{H}$ are perpendicular to radius r at any point A on the circular path, the angle θ is zero between them at all points. Also since all the points on the circular path are equidistant from the current carrying conductor is constant at all points on the circle

$$\oint \vec{H} \cdot \vec{dl} = \left|\vec{H}\right| \oint \vec{dl} = \left|\vec{H}\right| 2\pi r = I \quad \text{or} \quad \left|\vec{H}\right| = \frac{I}{2\pi r}$$

Magnetic Circuits

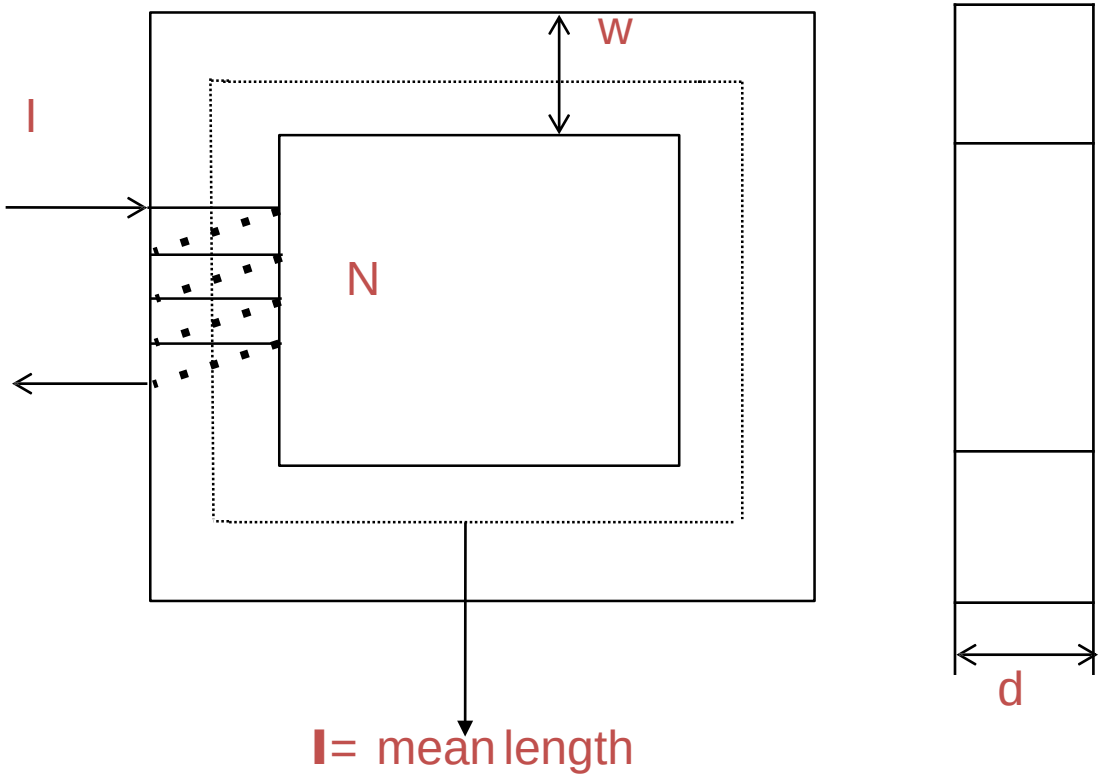
- They are basically ferromagnetic structures(mostly Iron, Cobalt, Nickel alloys and compounds) with coils wound around them

- Because of high permeability most of the magnetic flux is confined within the magnetic circuit

- Thus $\vec{H}$ is always aligned with $\vec{dl}(\theta = 0)$

- Examples: Transformers, Actuators, Electromagnets, Electric Machines

Magnetic Circuits (1)



Magnetic Circuits (2)

F =NI= Magneto Motive Force or MMF = # of turns * Current passing through it

F = NI = Hℓ (why!)

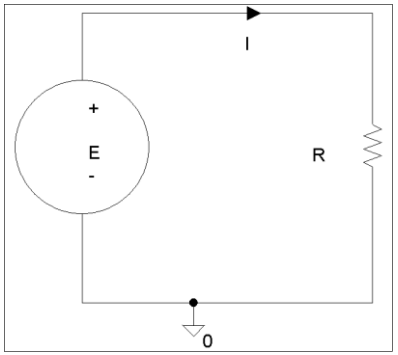
or $\frac{B}{\mu}\ell = NI$ or $\frac{\Phi}{\mu A}\ell = NI$

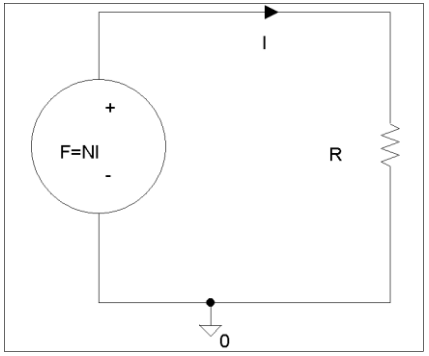
or $\Phi = \frac{NI}{\ell/(\mu A)}$

or $\Phi = \frac{NI}{\Re}$

$\Re$ = ***Reluctance*** of magnetic path

Analogy Between Magnetic and Electr Circuits





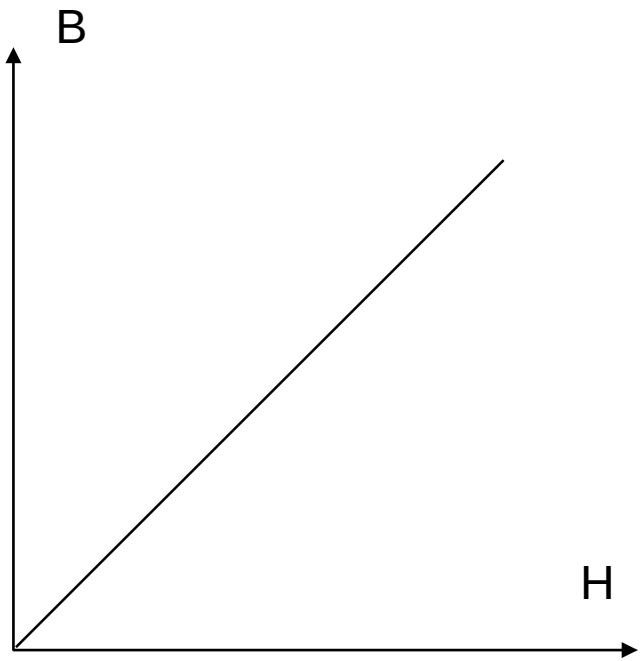
F =MMF is analogous to Electromotive force (EMF) =E

Φ = Flux is analogous to I = Current

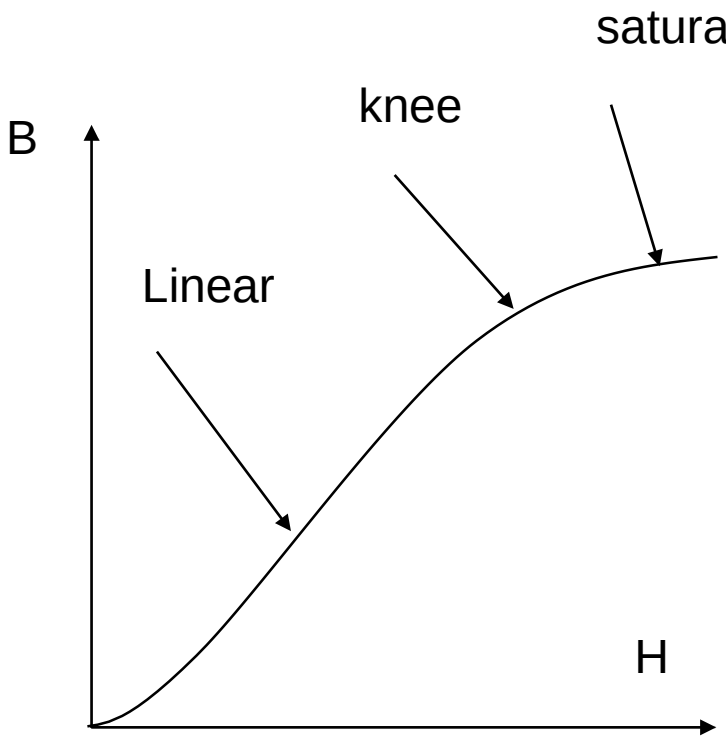
$\mathfrak{R}$ = Reluctance is analogous to R = Resistance

P = Permeance $= \frac{1}{\mathfrak{R}}$ = Analogous to conductance $G = \frac{1}{R}$

Magnetization Curves



Magnetization curve
(linear) (Ideal)



Magnetization curve
(non-linear) (Actual)
(see also Fig. 1.6 in the text)

Magnetization Curves(2)

•One can linearize magnetic circuits by including air-gaps

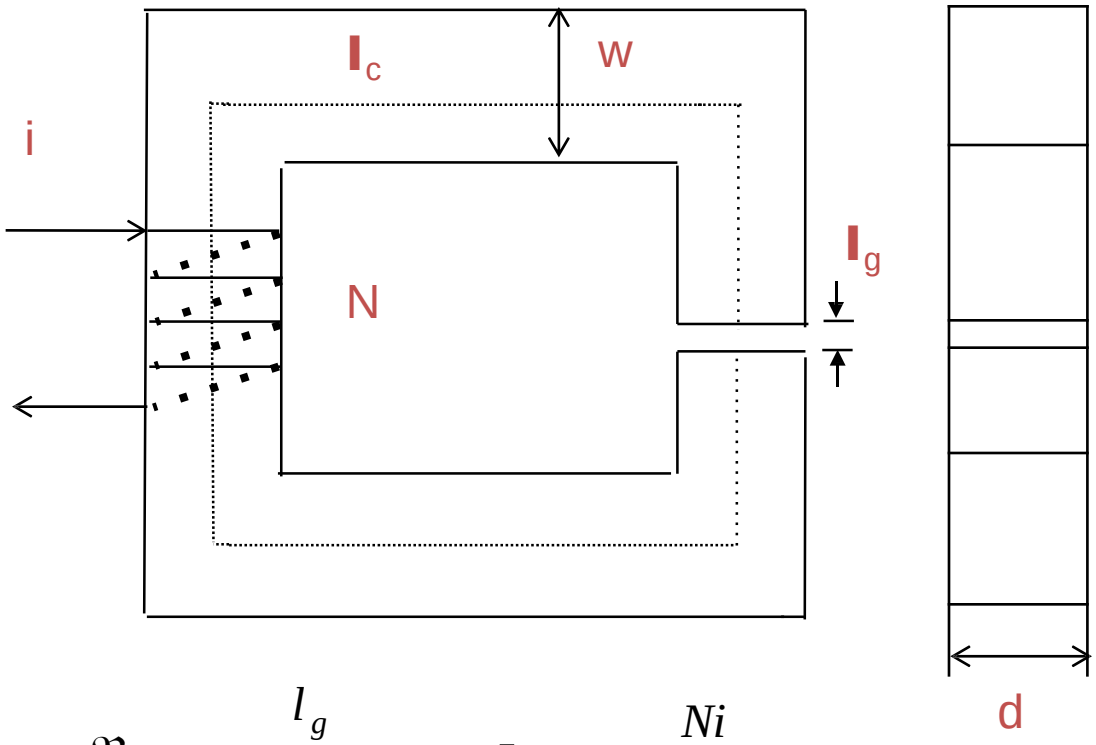
•However that would cause a large increase in ampere-turn requirements.

Ex: Transformers don‘t have air-gaps. They have very little magnetizing current (5% of full load)

Induction motors have air-gaps. They have large magnetizing current (30-50%)

Question: why induction motors have air –gap and transformers don‘t?

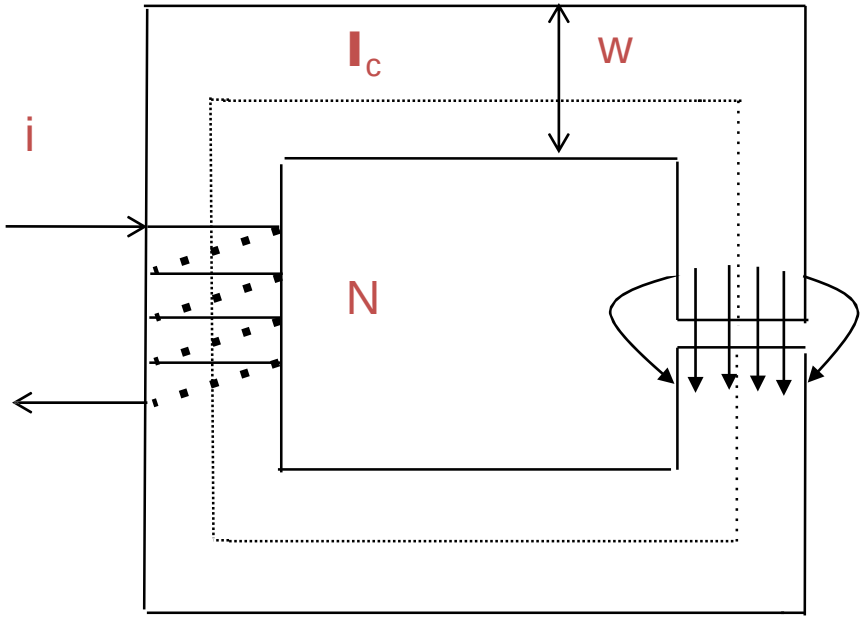
Magnetization Circuits with Air-gap



$$\Re_c = \frac{l_c}{\mu_c A_c} \qquad \Re_g = \frac{l_g}{\mu_g A_g} \qquad \Phi = \frac{Ni}{\Re_c + \Re_g}$$

$$Ni = H_c l_c + H_g l_g \qquad A_c = A_g = wd \quad (Neglecting \quad fringing \quad)$$

Fringing

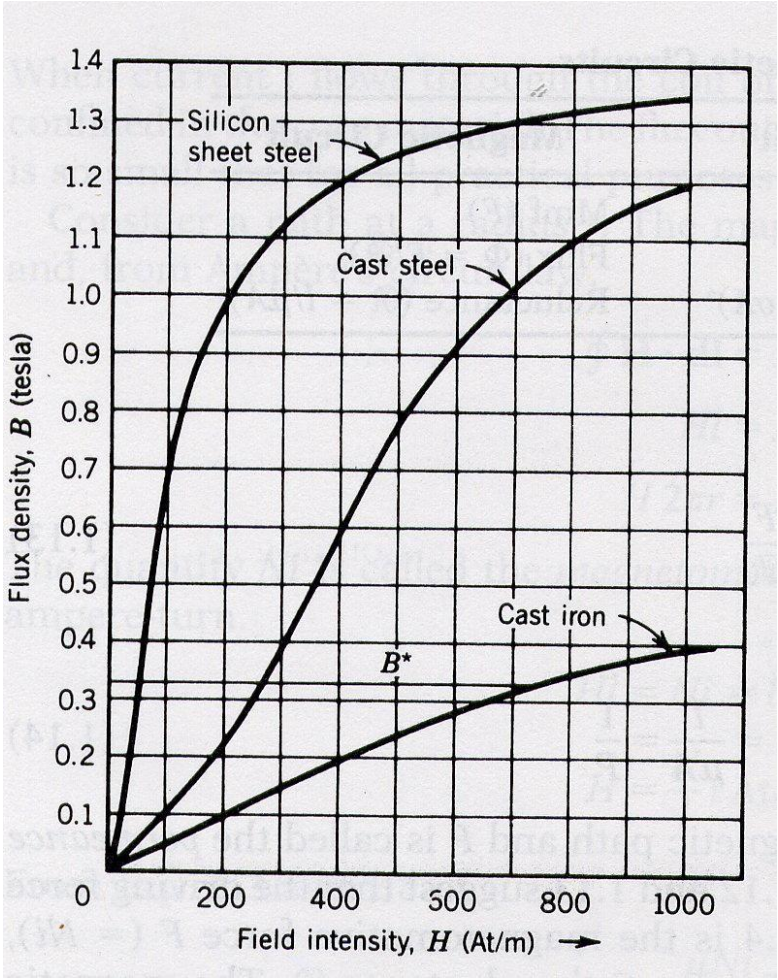


With large air-gaps, flux tends to leak outside the air –gap. This is called fringing which increases the effective flux area. One way to approximate this increase is:

$$w_n = w + l_g ; d_n = d + l_g ; A_{gn} = w_n d_n$$

Example of Magnetic Circuits On Greenboard

Magnetization Curves (for examples)



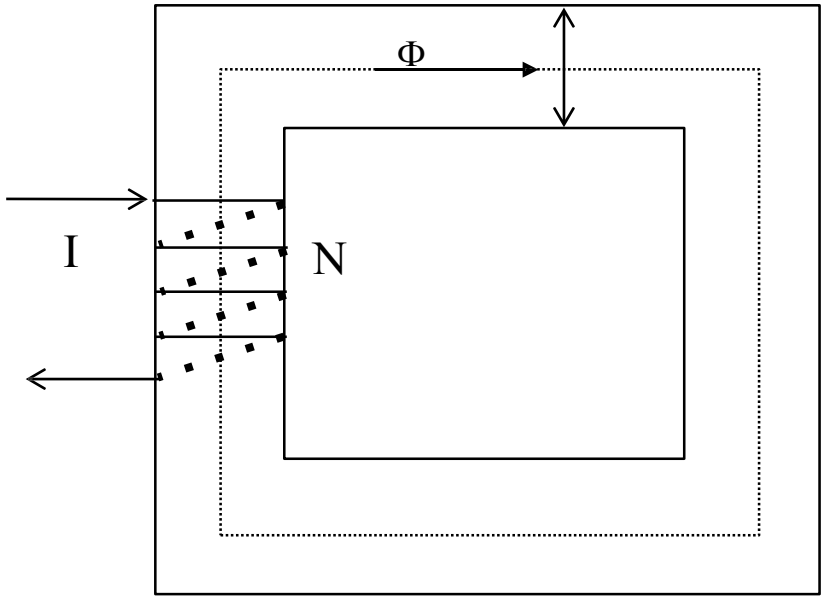
Inductance(L)

Definition: Flux Linkage(λ) per unit of current(I) in a magnetic circuit

$$L = \frac{\lambda}{I} = \frac{N\Phi}{I}$$

$$\Phi = \frac{NI}{\Re}$$

$$\therefore L = \frac{N^2}{\Re}$$



Thus inductance depends on the geometry of construction

Example of Inductances On Greenboard

How to find exact Inductances with magnetic circuit with finite thickness (say a torroid with finite thickness)

see problem 1.16

Faraday‘s law of Electromagnetic Induction

The EMF (Electromotive Force) induced in a magnetic circuit is Equal to the rate of change of flux linked with the circuit

$$e = \frac{d\lambda}{dt} = \frac{d(N\Phi)}{dt} = N\frac{d\Phi}{dt}$$

$$\square \quad Li = N\Phi$$

$$\therefore e = \frac{dLi}{dt} = L\frac{di}{dt}$$

Lenz‘s Law

The polarity of the induced voltage is given by Lenz‘s law

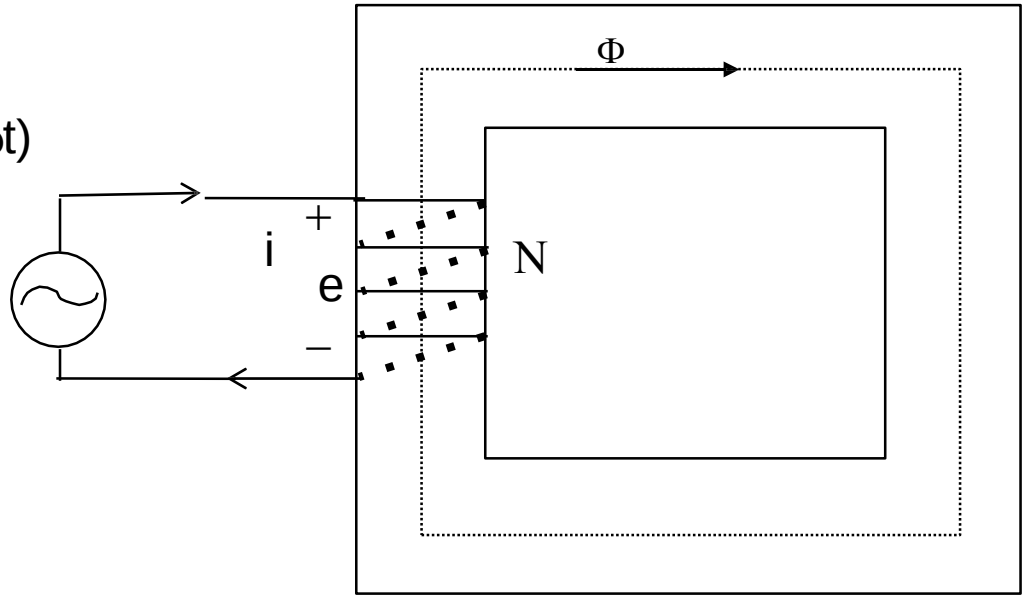
The polarity of the induced voltage will be such as to oppose the very cause to which it is due

Thus *sometimes* we write

$$\therefore e = -\frac{dLi}{dt} = -L\frac{di}{dt}$$

A precursor to Transformer

$V = V_m \cos(\omega t)$



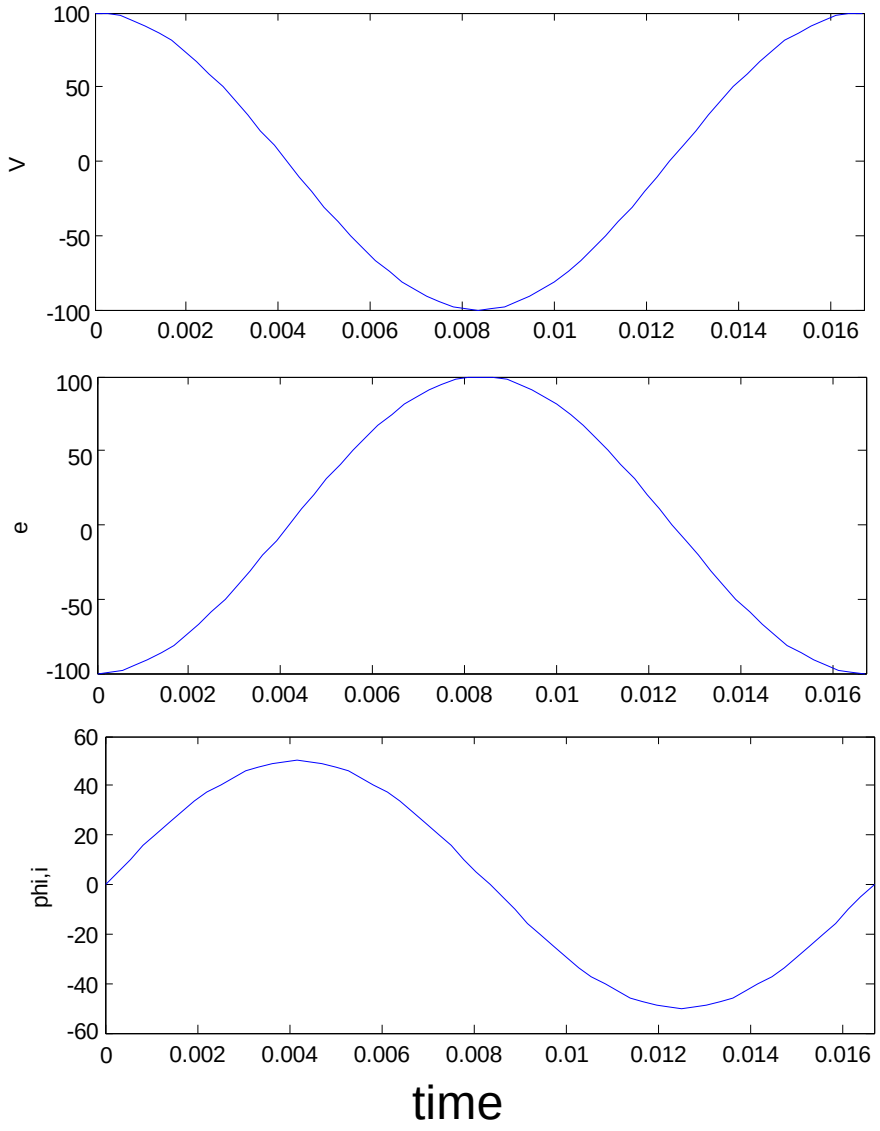
$\Phi = \Phi_m \sin(\omega t)$

Ideally

$$e = -N\frac{d\Phi}{dt} = -N\Phi_m \omega \cos(\omega t) = -E_m \cos(\omega t) = -V_m \cos(\omega t)$$

$$i = \frac{N\Phi}{L} = \frac{N\Phi_m \sin(\omega t)}{L} = I_m \sin(\omega t)$$

A Precursor to Transformer(2)



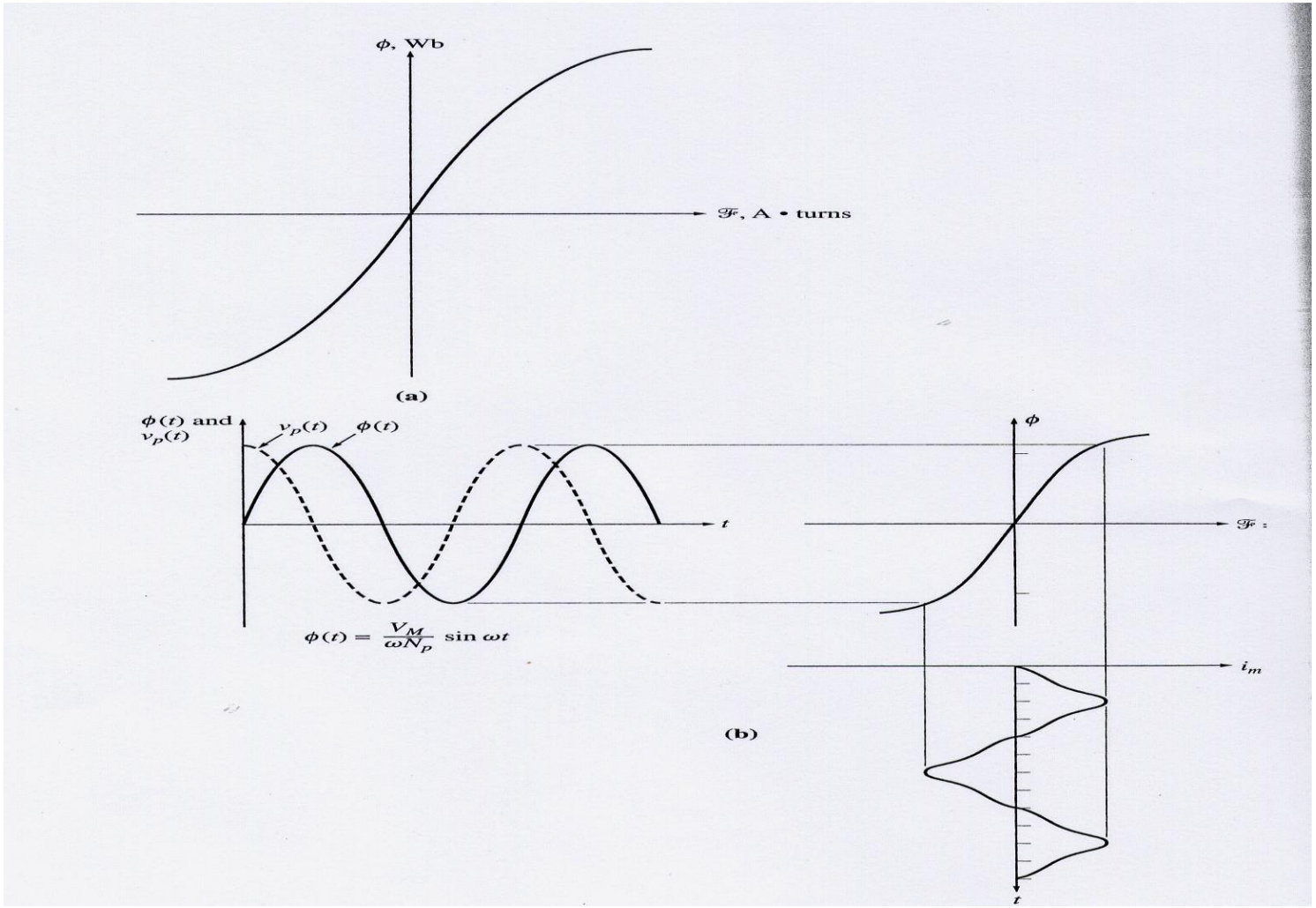
Example on excitation of magnetic circuit with sinusoidal flux On greenboard

Example on excitation of magnetic circuit with square flux on greenboard (Important for Switched Mode Power Supplies)

What will non-linearity in magnetic circuit lead to?

- It would cause distortion in current waveforms since by Faraday‘s and Lenz‘s law the induced voltage always has to balance out the applied voltage that happens to be sinusoidal

Sinusoidal voltage non-sinusoidal current



Iron Losses in Magnetic Circuit

There are two types of iron losses

a) Hysteresis losses

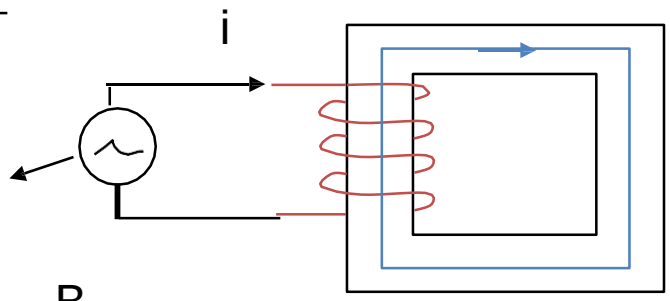
b) Eddy Current Losses

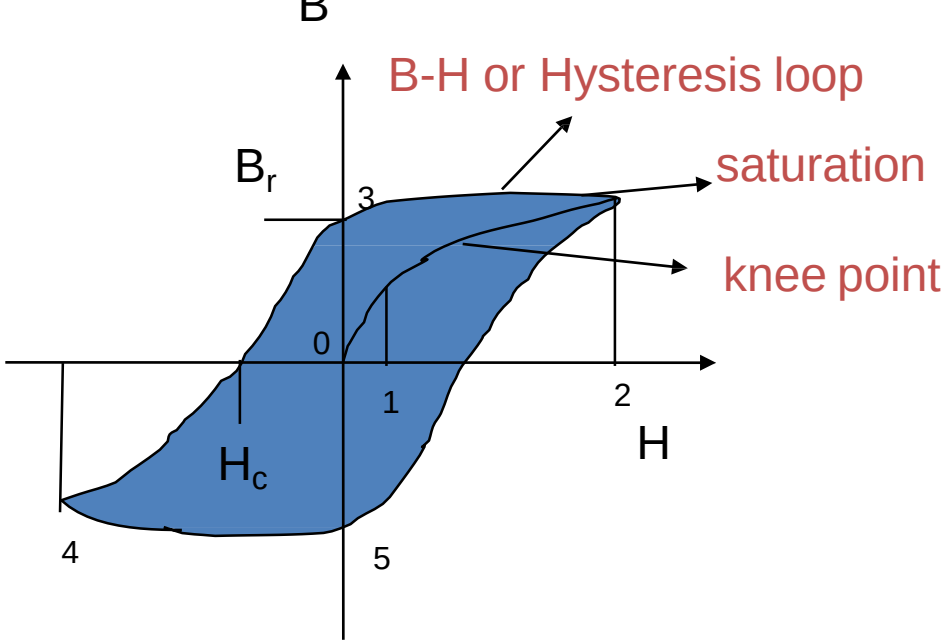
Total iron loss is the sum of these two losses

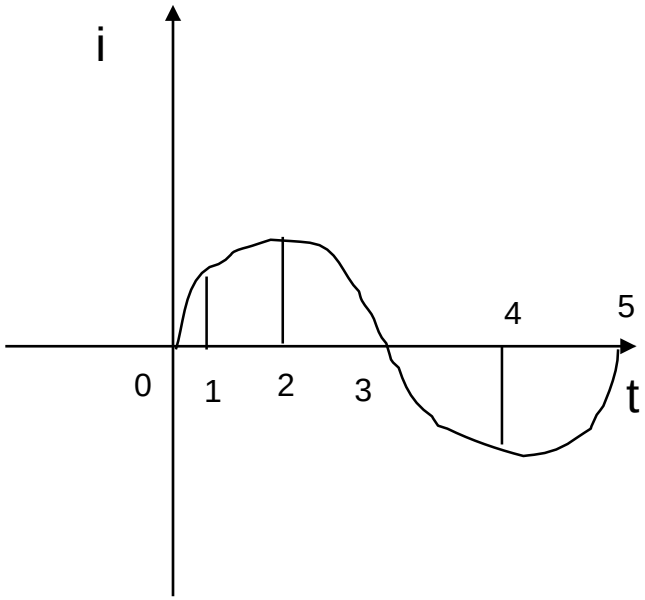
Hysteresis losses

$$f = \frac{1}{T}$$

f =frequency
of sine source







B_r = Retentive flux density (due to property of retentivity)
H_c= Coercive field intensity (due to property of coercivity)

Hysteresis losses (2)

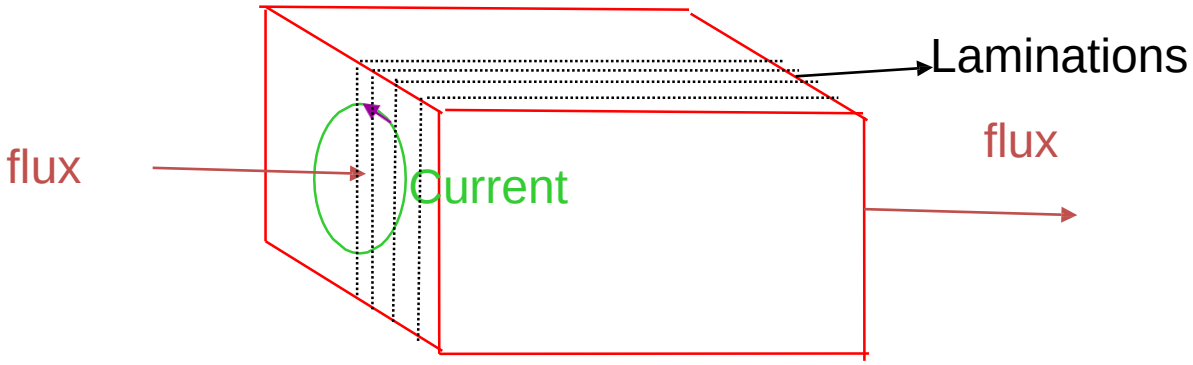
- The lagging phenomenon of B behind H is called hysteresis
- The tip of hysteresis loops can be joined to obtain the magnetization characteristics
- In each of the current cycle the energy lost in the core is proportional to the area of the B-H loop
- Energy lost/cycle = V_{core}

$$\oint HdB$$

- P_h = Hysteresis loss = $f V_{core} \oint HdB = k_h B^n_{max} f$

k_h = Constant, n = 1.5-2.5, B_{max}= Peak flux density

Eddy current loss



Because of time variation of flux flowing through the magnetic material as shown, current is induced in the magnetic material, following Faraday's law. This current is called eddy current.
The direction of the current is determined by Lenz's law. This current can be reduced by using laminated (thin sheet) iron structure, with Insulation between the laminations.

- P_e = Eddy current loss $= k_e B^2_{max} f$

k_e = Constant , B_{max}= Peak flux density

Permanent Magnets

- Alloys of Iron, Cobalt and Nickle

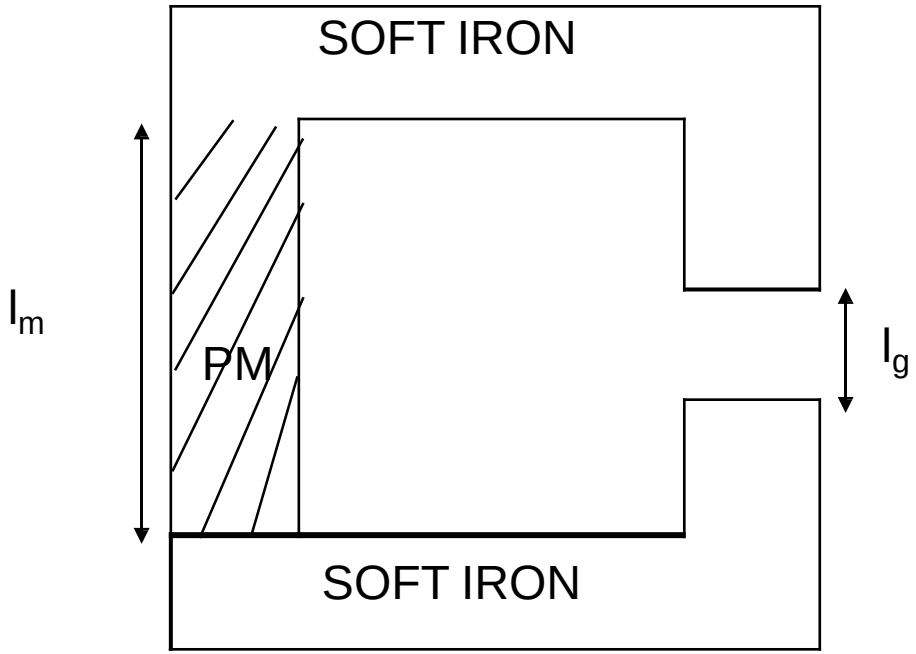
- Have large B-H loops, with large B_r and $-H_c$

- Due to heat treatment becomes mechanically hard and are thus called HARD IRON

- Field intensity is determined by the coercive field required to demagnetize it

- Operating points defined by B_m,H_m in the second quadrant of the B-H loop

Using Permanent Magnets for providing magnetic field



Designing Permanent Magnets

- The key issue here is to minimize the volume V_m of material required for setting up a required B_g in a given air gap

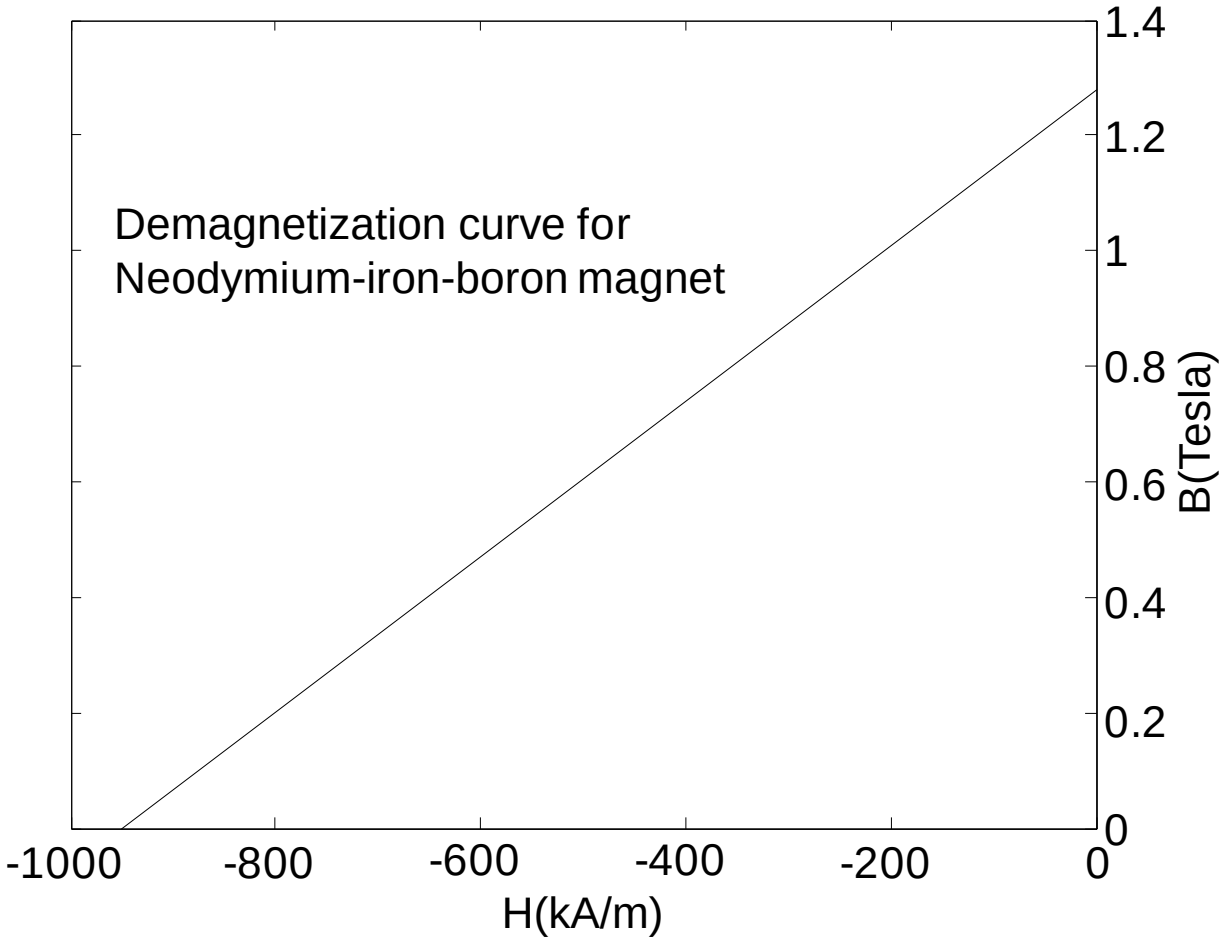
- It can be shown that $V_m = B_g^2 V_g / \mu_o B_m H_m$ (see derivation in text) where $V_g = A_g l_g$ Volume of air-gap, l_g = length of air-gap, A_g =area of air-gap

- Thus by maximizing B_m, H_m product V_m can be minimized

- Once B_m, H_m at the maximum B_m, H_m product point are known, l_m =length of permanent magnet, A_m =area of permanent magnet can be found as

- $l_m = -l_g H_g / H_m$ (applying ampère‘s law),
- $A_m = B_g A_g / B_m$ (same flux flows through PM as well as air-gap)

Finding the maximum product point



Finding the maximum product point (

B= mH+c, m and c are constants.

To find maximum BH product, we need to differentiate

$$BH=mH^2+cH;$$

and set it equal to 0. Thus we get

H_m=-c/2m. and B_m =c/2

Finding the maximum product point (

Answer:

B_m=0.64 T, H_m = -475 kA/m

UNIT- V

NETWORK THEOREMS

Introduction

This chapter introduces important fundamental theorems of network analysis. They are the

- **Superposition theorem**
- **Thévenin's theorem**
- **Norton's theorem**
- **Maximum power transfer theorem**
- **Substitution Theorem**
- **Millman's theorem**
- **Reciprocity theorem**

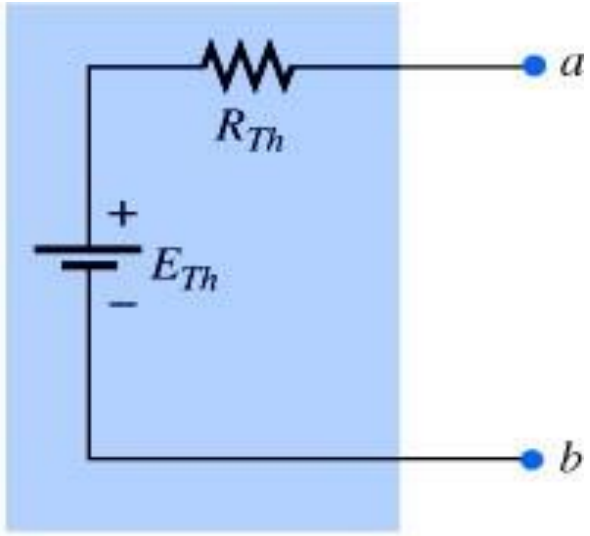
Superposition Theorem

Used to find the solution to networks with two or more sources that are not in series or parallel. The current through, or voltage across, an element in a network is equal to the algebraic sum of the currents or voltages produced independently by each source. Since the effect of each source will be determined independently, the number of networks to be analyzed will equal the number of sources.

Superposition Theorem

- The total power delivered to a resistive must be determined using the total current through or the total voltage across the element and cannot be determined by a simple sum of the power levels established by each source.

Thévenin's Theorem



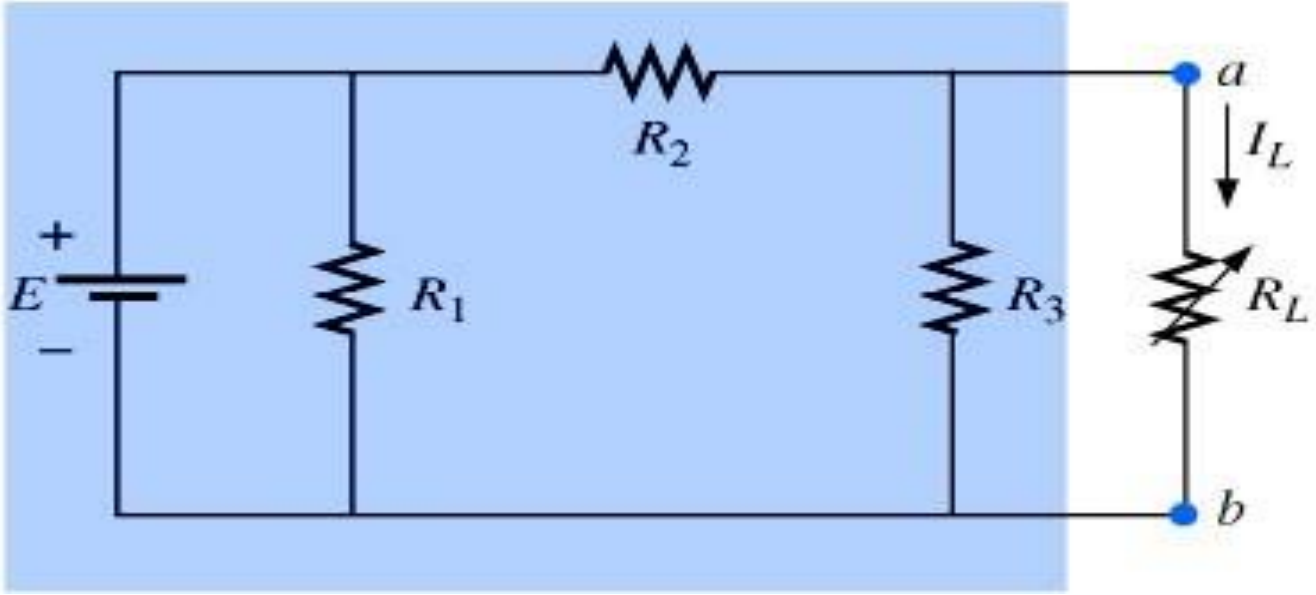
Thévenin's Theorem

Reduce the number of components required to establish the same characteristics at the output terminals.Investigate the effect ofchanging a particular component on the behavior of a network without having to analyze the entire network after each change.

Thévenin's Theorem

- Procedure to determine the proper values of R_{Th} and E_{Th}
- Preliminary

1. **Remove that portion of the network across which the Thévenin equation circuit is to be found. In the figure below, this requires that the load resistor R_L be temporarily removed from the network.**



Thévenin's Theorem

2. Mark the terminals of the remaining two-terminal network. (The importance of this step will become obvious as we progress through some complex networks.)

R_{Th}:

3. Calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)

Thévenin's Theorem

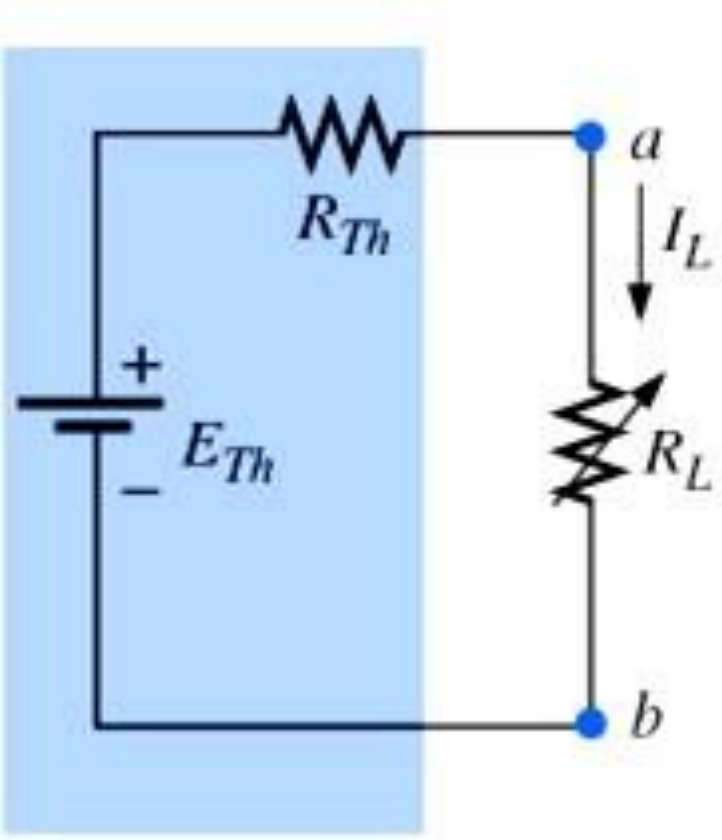
E_{Th}:

4. Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that will lead to the most confusion and errors. In all cases, keep in mind that it is the open-circuit potential between the two terminals marked in step 2.)

Thévenin's Theorem

Conclusion:

5. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the resistor R_L between the terminals of the Thévenin equivalent circuit.



Thévenin's Theorem

Experimental Procedures

- Two popular experimental procedures for determining the parameters of the Thévenin equivalent network:

Direct Measurement of E_{Th} and R_{Th}

For any physical network, the value of E_{Th} can be determined experimentally by measuring the open-circuit voltage across the load terminals.

The value of R_{Th} can then be determined by completing the network with a variable resistance R_L.

Thévenin's Theorem

Measuring V_{OC} and I_{SC} The Thévenin voltage is again determined by measuring the open-circuit voltage across the terminals of interest; that is,

$E_{Th} = V_{OC}$. To determine R_{Th}, a short-circuit condition is established across the terminals of interest and the current through the short circuit (I_{sc}) is measured with an ammeter.

Using Ohm's law:

$$R_{Th} = V_{oc} / I_{sc}$$

Norton's Theorem

Norton's theorem states the following:

Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current and a parallel resistor.

The steps leading to the proper values of I_N and R_N.

Preliminary steps:

1. Remove that portion of the network across which the Norton equivalent circuit is found.
2. Mark the terminals of the remaining two-terminal network.

Norton's Theorem

- Finding R_N:

3. Calculate R_N by first setting all sources to zero (voltage sources are replaced with short circuits, and current sources with open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) Since $R_N = R_{Th}$ the procedure and value obtained using the approach described for Thévenin's theorem will determine the proper value of R_N.

Norton's Theorem

Finding I_N :

4. Calculate I_N by first returning all the sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.

Conclusion:

5. Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Maximum Power Transfer Theorem

A load will receive maximum power from a network when its total resistive value is exactly equal to the Thévenin resistance of the network applied to the load. That is,

$$R_L = R_{Th}$$

Maximum Power Transfer Theorem

For loads connected directly to a dc voltage supply, maximum power will be delivered to the load when the load resistance is equal to the internal resistance of the source; that is, when:

$$R_L = R_{int}$$

Millman's Theorem

Any number of parallel voltage sources can be reduced to one.

This permits finding the current through or across R_L without having to apply a method such as mesh analysis, nodal analysis, superposition and so on.

1. Convert all voltage sources to current sources.
2. Combine parallel current sources.
3. Convert the resulting current source to a voltage source and the desired single-source network is obtained.

Substitution Theorem

The substitution theorem states: If the voltage across and the current through any branch of a dc bilateral network is known, this branch can be replaced by any combination of elements that will maintain the same voltage across and current through the chosen branch. Simply, for a branch equivalence, the voltage and current must be the same.

Reciprocity Theorem

The reciprocity theorem is applicable only to single-source networks and states the following:

The current ***I*** in any branch of a network, due to a single voltage source ***E*** anywhere in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current ***I*** was originally measured.

- The location of the voltage source and the resulting may be interchanged without a change in current

THANK YOU