CONTROL SYSTEMS

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Poles and Zeros and Transfer Functions

Transfer Function: A transfer function is defined as the ratio of the Laplace transform of the output to the input with all initial conditions equal to zero. Transfer functions are defined only for linear time invariant systems.

Considerations: Transfer functions can usually be expressed as the ratio of two polynomials in the complex variable, s.

Factorization: A transfer function can be factored into the following form.

$$G(s) = \frac{K(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ldots(s+p_n)}$$

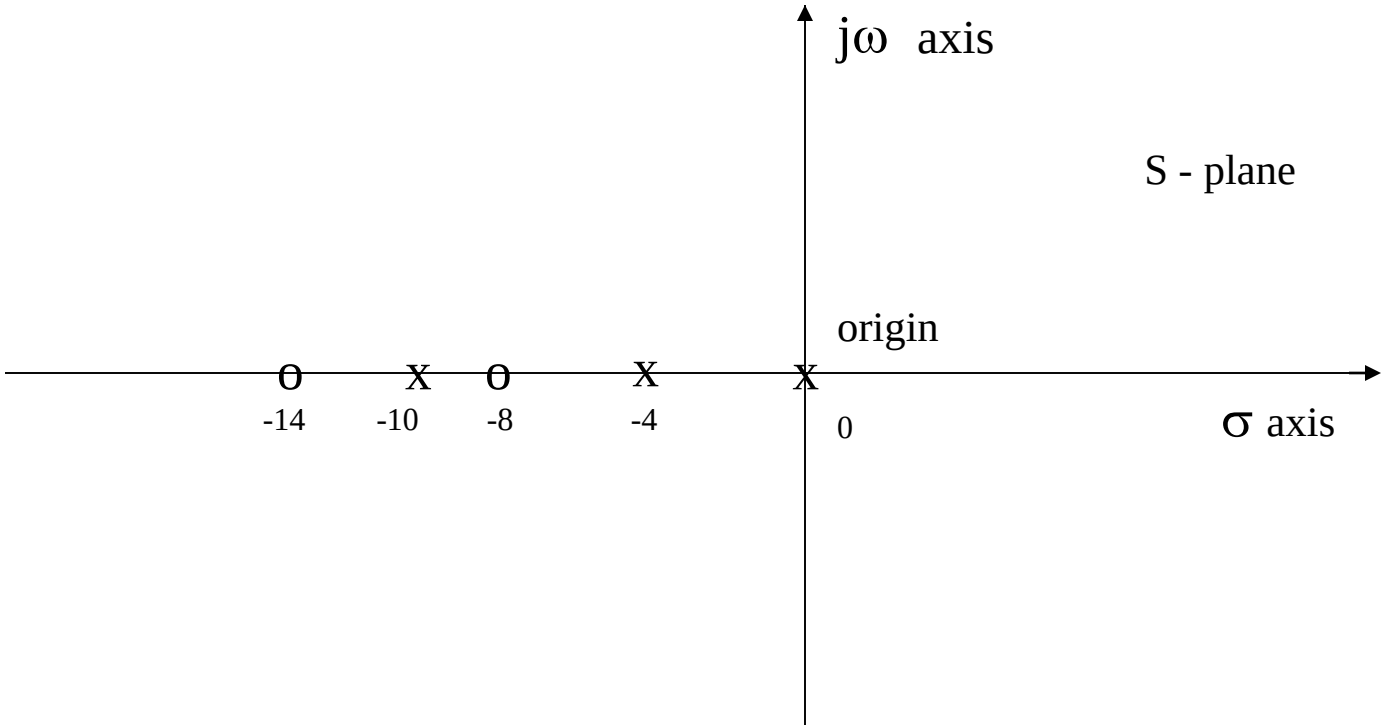
The roots of the numerator polynomial are called zeros.

The roots of the denominator polynomial are called poles.

Poles, Zeros and the S-Plane

An Example: You are given the following transfer function. Show the poles and zeros in the s-plane.

$$G(s) = \frac{(s+8)(s+14)}{s(s+4)(s+10)}$$



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Poles, Zeros and Bode Plots

Characterization:

Considering the transfer function of the previous slide. We note that we have 4 different types of terms in the previous general form: These are:

$$K_B,\ \frac{1}{s},\ \frac{1}{(s/p\ +\ 1)},\ \ (s/z+1)$$

Expressing in dB:

Given the tranfer function:

$$G(jw)=\frac{K_B(jw/z+1)}{(jw)(jw/p+1)}$$

$$20\log|G(jw|=20\log K_B+20\log|(jw/z+1)|-20\log|\ jw|-20\log|\ jw/p+1|$$

Poles, Zeros and Bode Plots

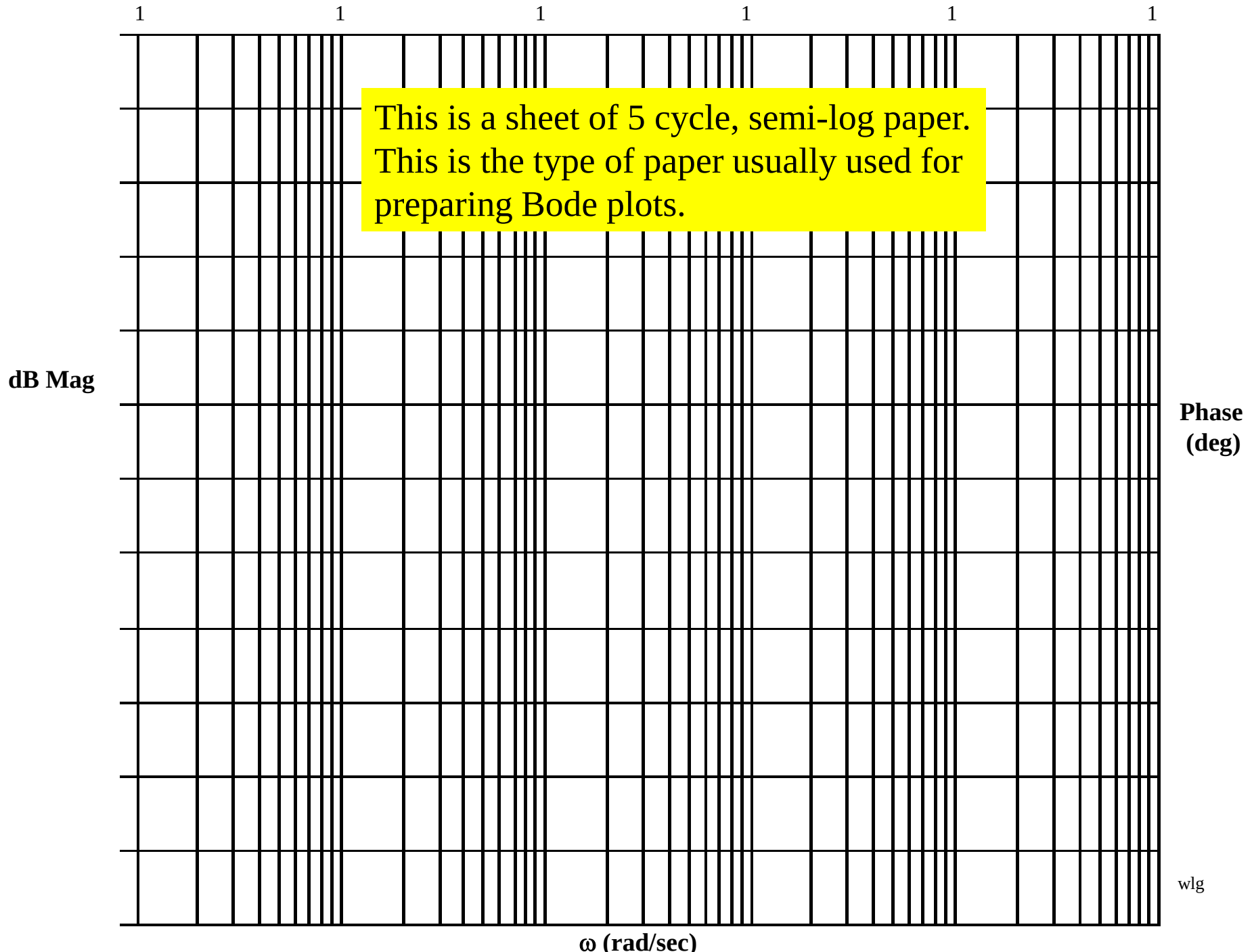
Mechanics: We have 4 distinct terms to consider:

$20\log K_B$

$20\log|(jw/z + 1)|$

$-20\log|jw|$

$-20\log|(jw/p + 1)|$

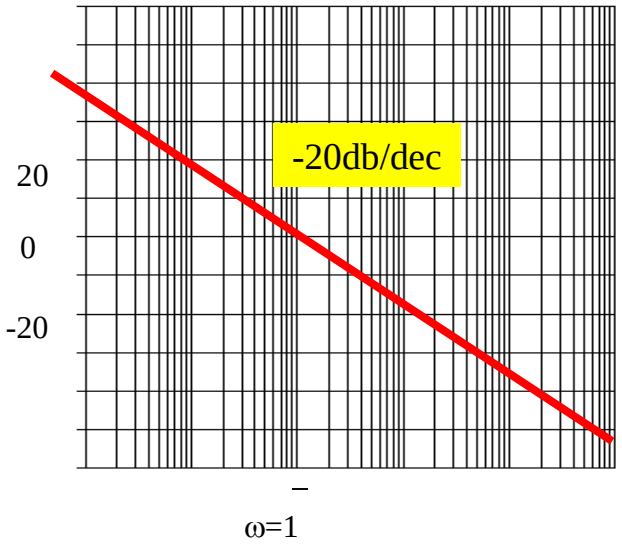

1
1
1
1
1
1
This is a sheet of 5 cycle, semi-log paper. This is the type of paper usually used for preparing Bode plots.
dB Mag
Phase (deg)
wlg
ω (rad/sec)

Poles, Zeros and Bode Plots

Mechanics:

The gain term, $20\log K_B$, is just so many dB and this is a straight line on Bode paper, independent of omega (radian frequency).

The term, $-20\log|jw| = -20\log w$, when plotted on semi-log paper is a straight line sloping at -20dB/decade. It has a magnitude of 0 at w = 1.



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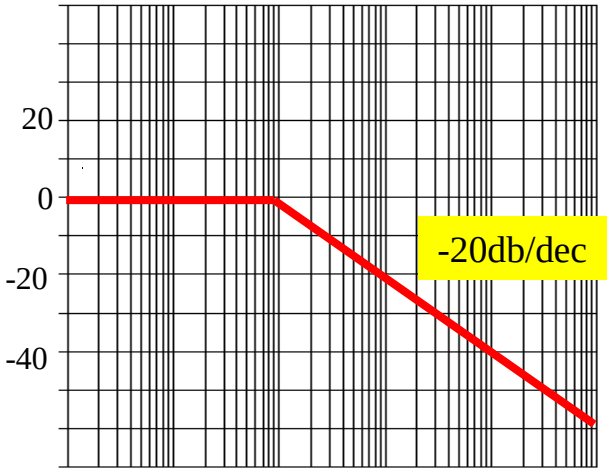
Poles, Zeros and Bode Plots

Mechanics:

The term, - 20log|(jw/p + 1), is drawn with the following approximation: If w < p we use the approximation that –20log|(jw/p + 1)| = 0 dB, a flat line on the Bode. If w > p we use the approximation of –20log(w/p), which slopes at -20dB/dec starting at w = p. Illustrated below. It is easy to show that the plot has an error of -3dB at w = p and – 1 dB at w = p/2 and w = 2p. One can easily make these corrections if it is appropriate.



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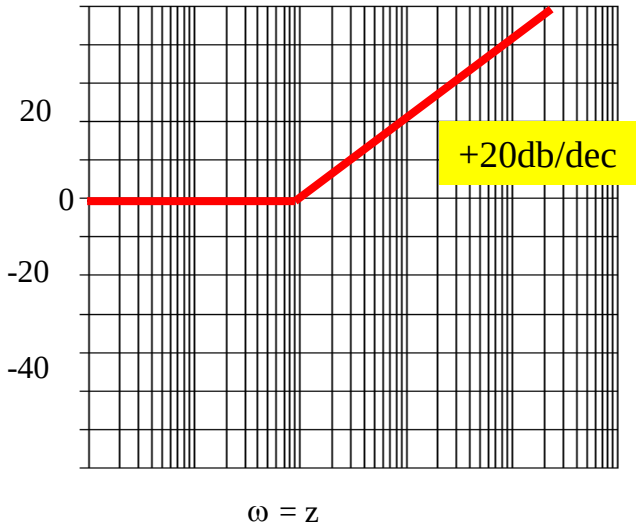
Poles, Zeros and Bode Plots

Mechanics:

When we have a term of 20log|(jw/z + 1)| we approximate it be a straight line of slop 0 dB/dec when w < z. We approximate it as 20log(w/z) when w > z, which is a straight line on Bode paper with a slope of + 20dB/dec. Illustrated below.



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Example 1:

Given:

$$G(jw) = \frac{50,000(jw+10)}{(jw+1)(jw+500)}$$

First: Always, always, always get the poles and zeros in a form such that the constants are associated with the jw terms. In the above example we do this by factoring out the 10 in the numerator and the 500 in the denominator.

$$G(jw) = \frac{50,000x10(jw/10+1)}{500(jw+1)(jw/500+1)} = \frac{100(jw/10+1)}{(jw+1)(jw/500+1)}$$

Second: When you have neither poles nor zeros at 0, start the Bode at $20\log_{10}K = 20\log_{10}100 = 40$ dB in this case.

Example 1: (continued)

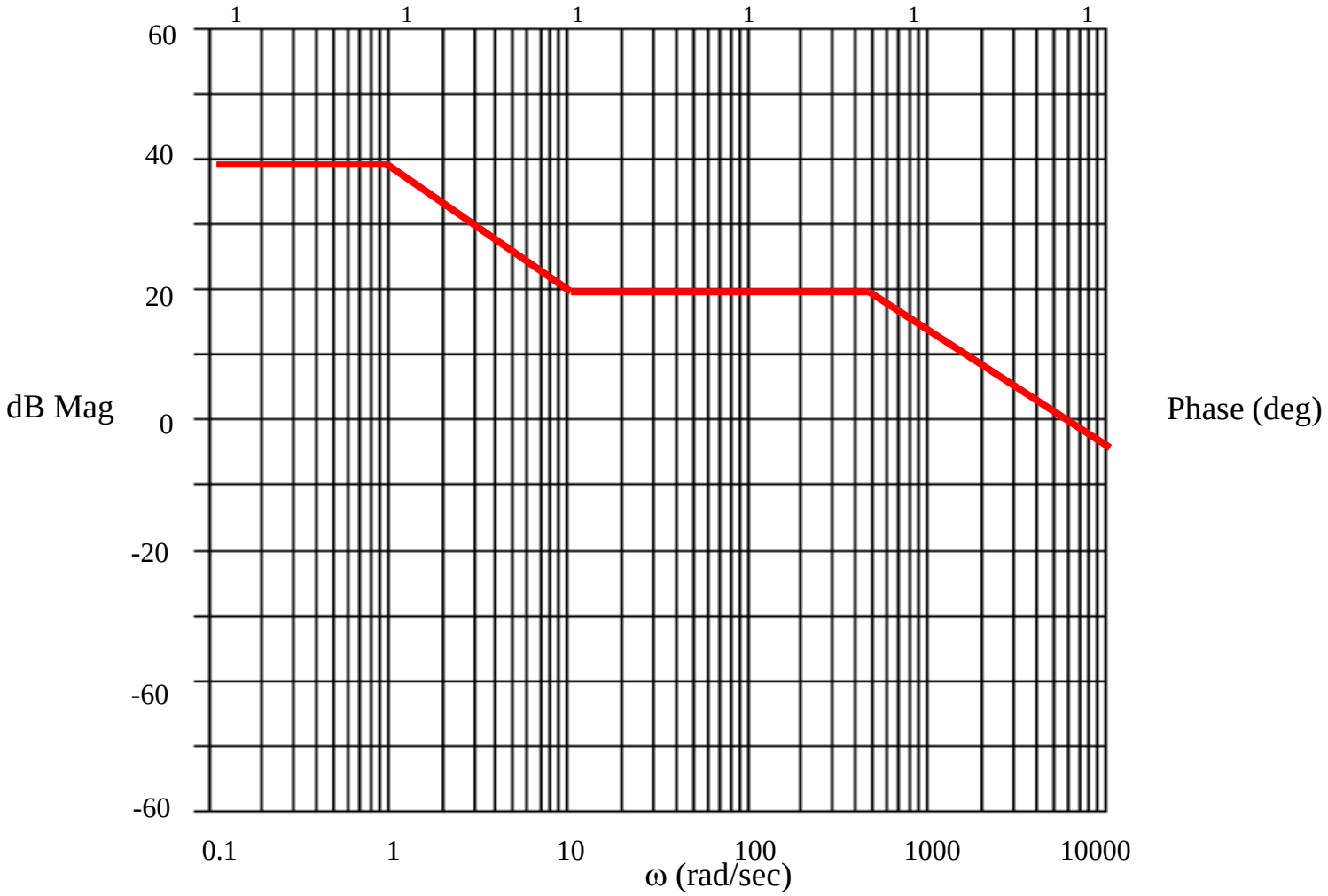
Third: Observe the order in which the poles and zeros occur.
This is the secret of being able to quickly sketch the Bode.
In this example we first have a pole occurring at 1 which causes the Bode to break at 1 and slope – 20 dB/dec.
Next, we see a zero occurs at 10 and this causes a slope of +20 dB/dec which cancels out the – 20 dB/dec, resulting in a flat line (0 db/dec). Finally, we have a pole that occurs at w = 500 which causes the Bode to slope down at – 20 dB/dec.

We are now ready to draw the Bode.

Before we draw the Bode we should observe the range over which the transfer function has active poles and zeros. This determines the scale we pick for the w (rad/sec) at the bottom of the Bode.

The dB scale depends on the magnitude of the plot and experience is the best teacher here.

Bode Plot Magnitude for 100(1 + jw/10)/(1 + jw/1)(1 + jw/500)



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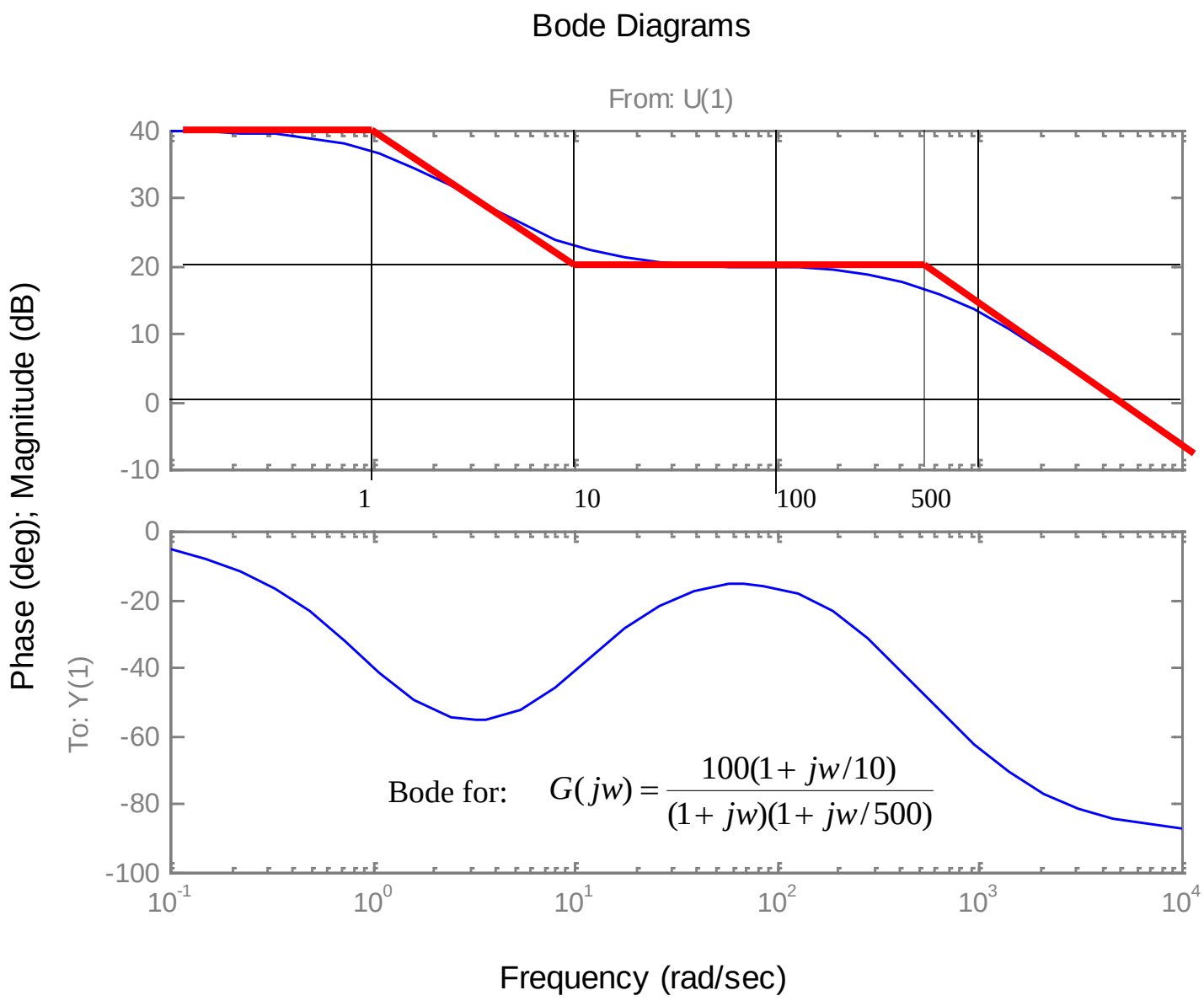
Using Matlab For Frequency Response

Instruction: We can use Matlab to run the frequency response for the previous example. We place the transfer function in the form:

$$\frac{5000(s+10)}{(s+1)(s+500)} = \frac{[5000s+50000]}{[s^2+501s+500]}$$

In the following slide, the resulting magnitude and phase plots (exact) are shown in light color (blue). The approximate plot for the magnitude (Bode) is shown in heavy lines (red). We see the 3 dB errors at the corner frequencies.


Bode Diagrams
From: U(1)
40
30
20
10
0
-10
1
10
100
500
0
-20
-40
-60
-80
-100
To: Y(1)
Phase (deg); Magnitude (dB)
Bode for: $G(jw) = \frac{100(1+jw/10)}{(1+jw)(1+jw/500)}$
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Frequency (rad/sec)

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Phase for Bode Plots

Comment:

Generally, the phase for a Bode plot is not as easy to draw or approximate as the magnitude. In this course we will use an analytical method for determining the phase if we want to make a sketch of the phase.

Illustration:

Consider the transfer function of the previous example. We express the angle as follows:

$$\angle G(jw) = \tan^{-1}(w/10) - \tan^{-1}(w/1) - \tan^{-1}(w/500)$$

We are essentially taking the angle of each pole and zero. Each of these are expressed as the $\tan^{-1}$(j part/real part)

Usually, about 10 to 15 calculations are sufficient to determine a good idea of what is happening to the phase.

Bode Plots

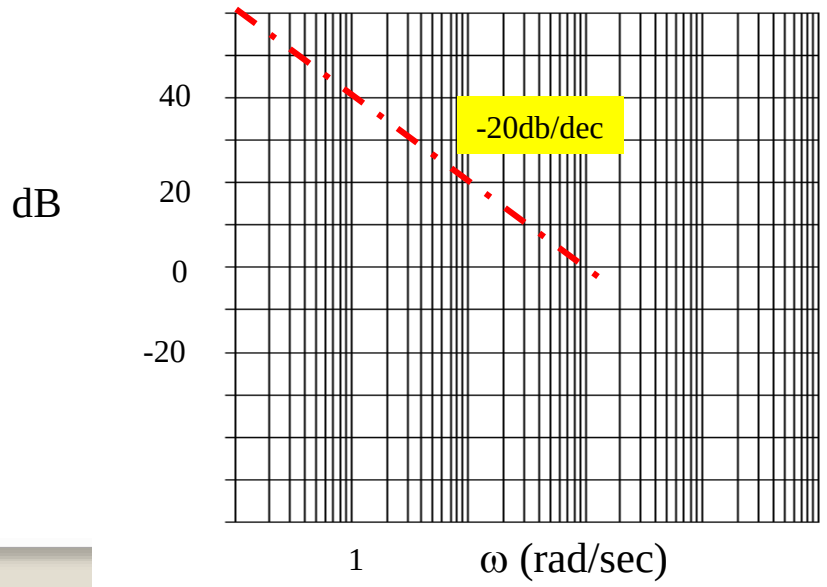
Example 2: Given the transfer function. Plot the Bode magnitude.

$$G(s)=\frac{100(1+s/10)}{s(1+s/100)^2}$$

Consider first only the two terms of

$$\frac{100}{jw}$$

Which, when expressed in dB, are; 20log100 – 20 logw. This is plotted below.

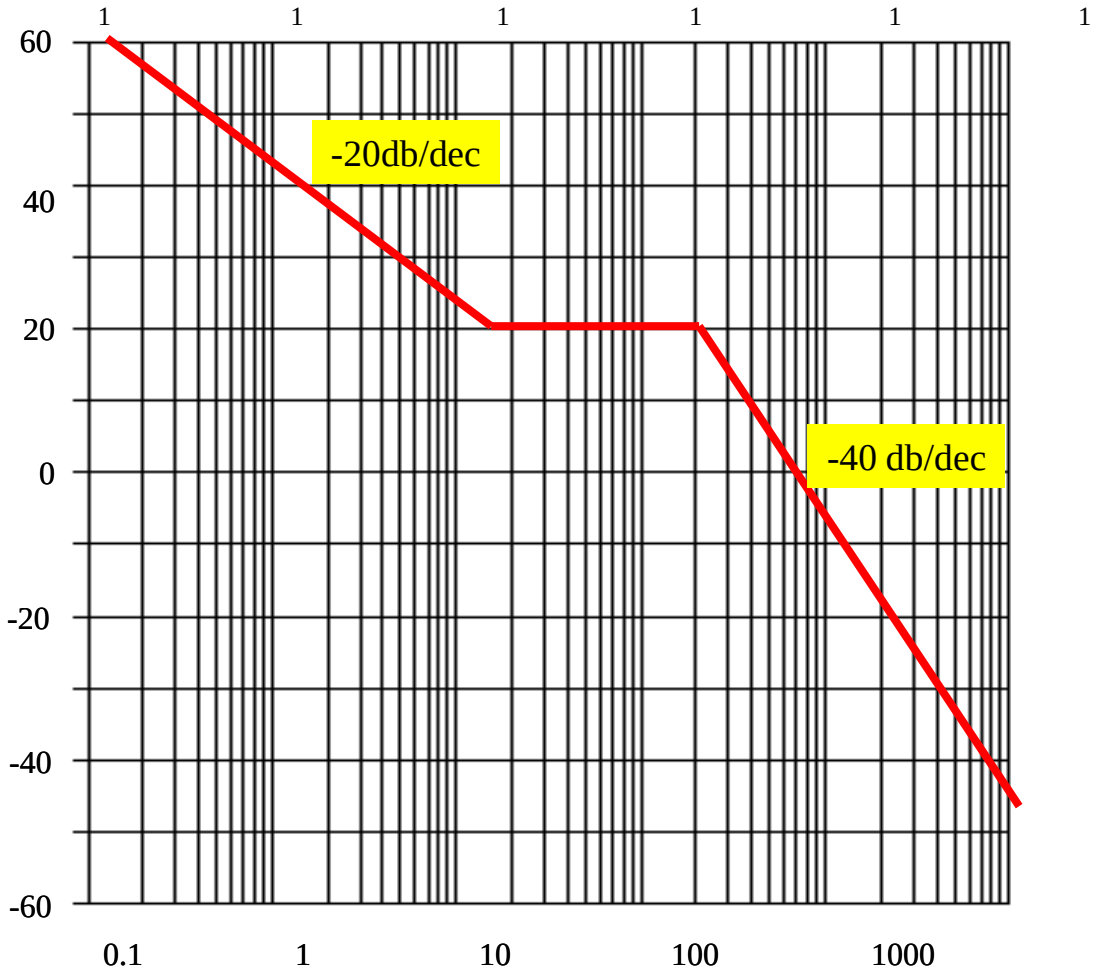


The ..... is a tentative line we use until we encounter the first pole(s) or zero(s) not at the origin.

Bode Plots

Example 2: (continued) The completed plot is shown below.

$$G(s)=\frac{100(1+s/10)}{s(1+s/100)^2}$$



dB Mag

Phase (deg)

$$G(s)=\frac{100(1+s/10)}{s(1+s/100)^2}$$

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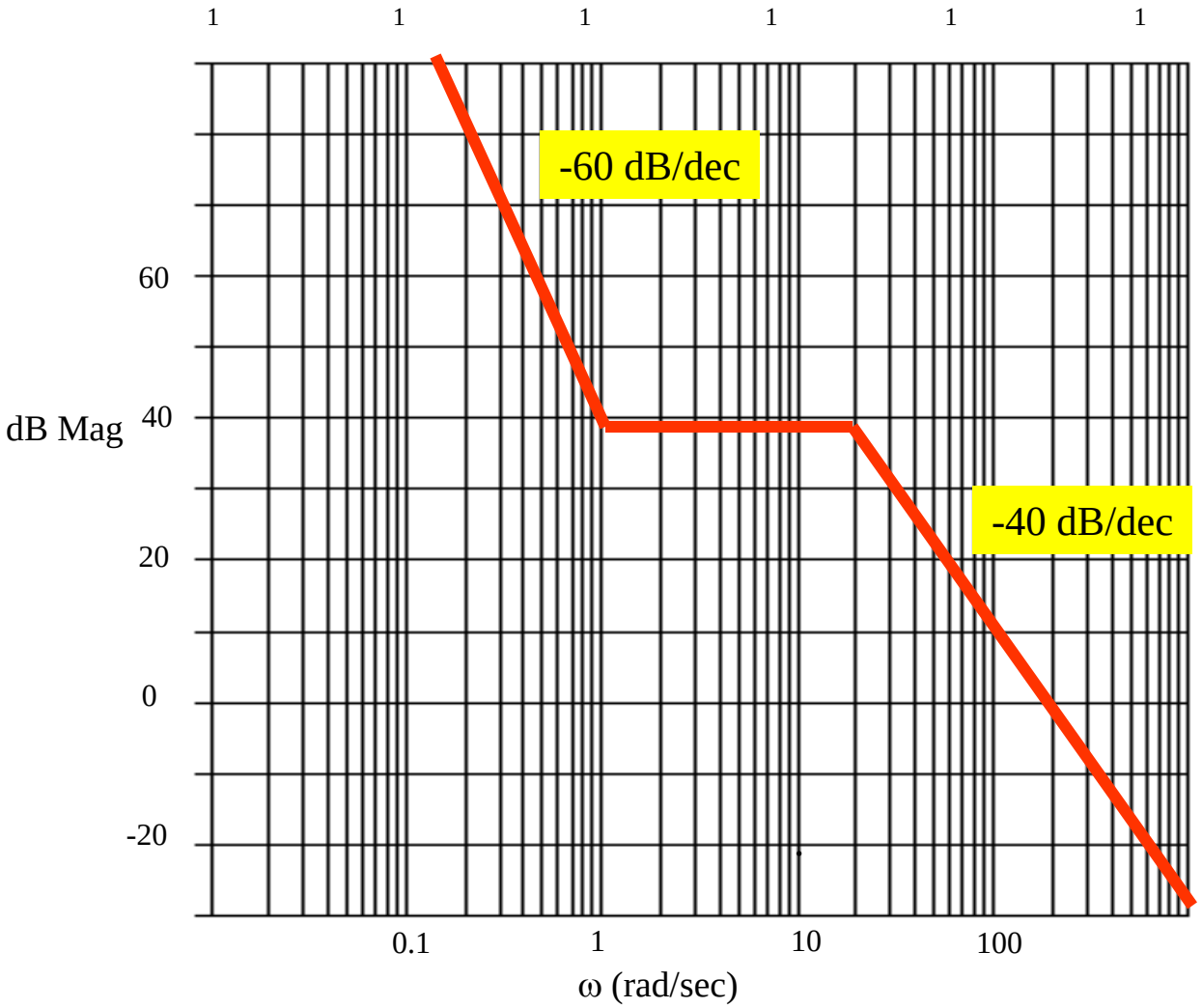
Bode Plots

Example 3:

Given:

$$G(s) = \frac{80(1 + jw)^3}{(jw)^3(1 + jw/20)^2}$$

20log80 = 38 dB



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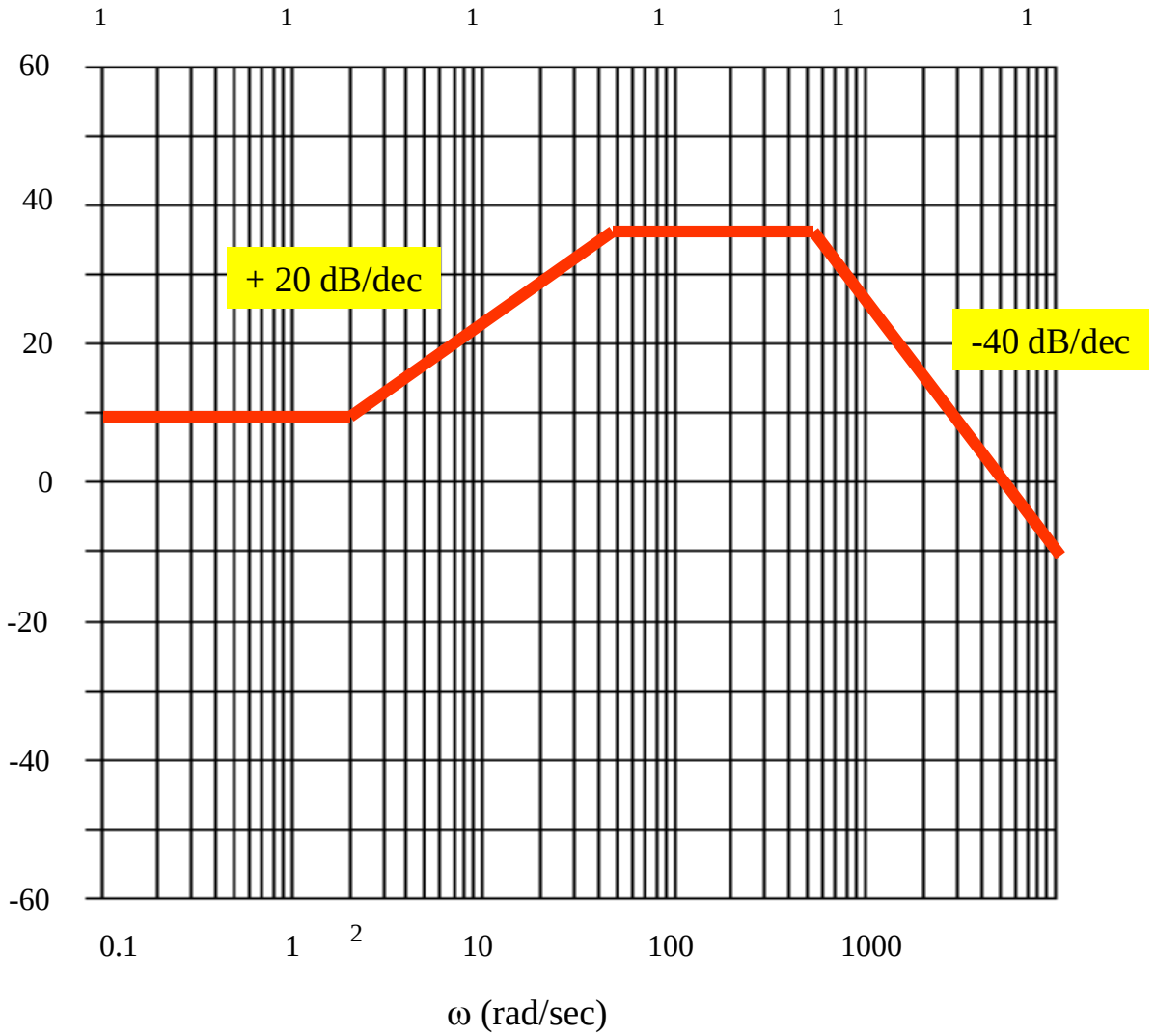
Bode Plots

Example 4:

Given:

$$G(jw) = \frac{10(1 - jw/2)}{(1 + j0.025w)(1 + jw/500)^2}$$



Sort of a low pass filter

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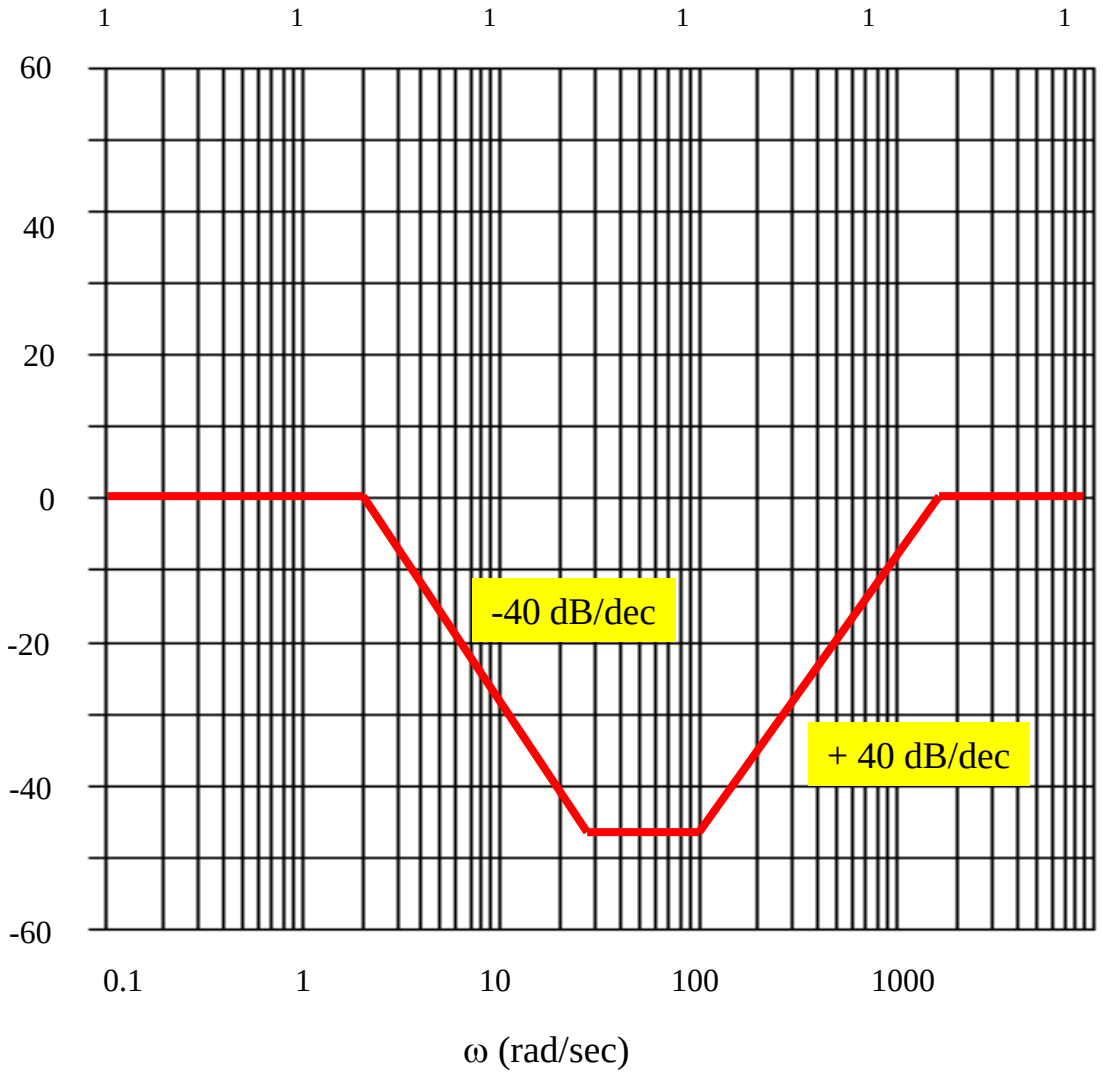
Bode Plots

Given:

$$G(jw)=\frac{(1+jw/30)^2(1+jw/100)^2}{(1+jw/2)^2(1+jw/1700)^2}$$

Example 5



dB Mag

Phase (deg)

Sort of a low pass filter

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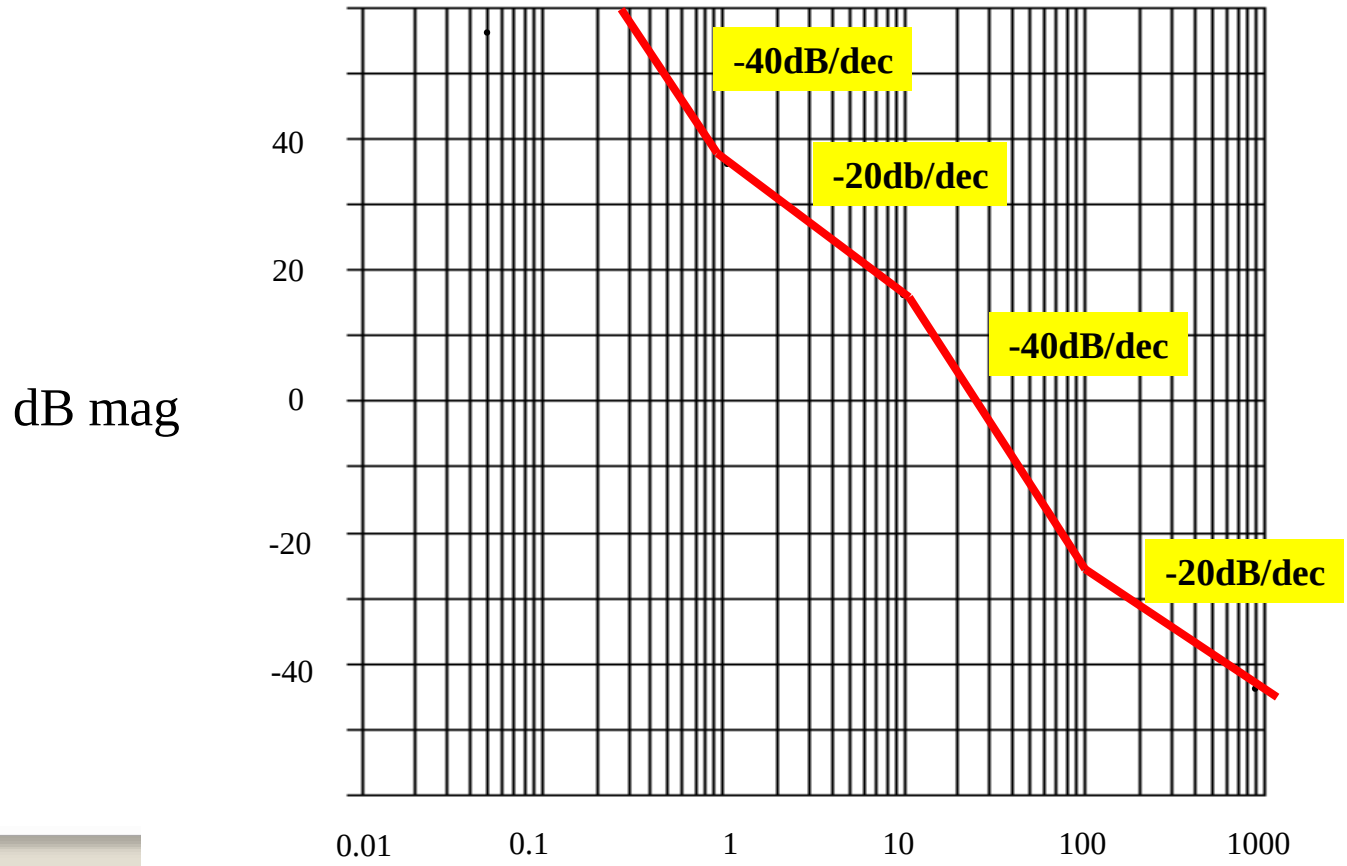
Bode Plots

Example 6

$$H(jw)=\frac{640(jw+1)(0.01jw+1)}{(jw)^2(jw+10)}=\frac{64(jw+1)(0.01jw+1)}{(jw)^2(0.1jw+1)}$$



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